Timing noise and glitches:

Limitations on pulsar timing for spacecraft navigation

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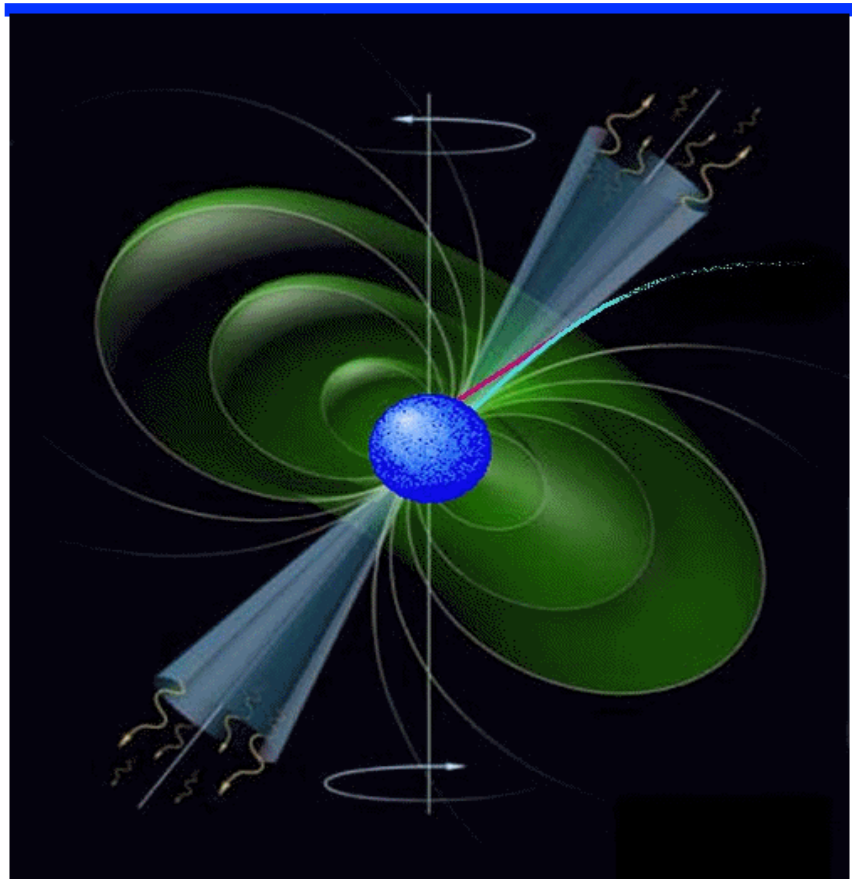
Secular Spin-down: physical model?



$$\dot{\Omega} = -K\Omega^n$$

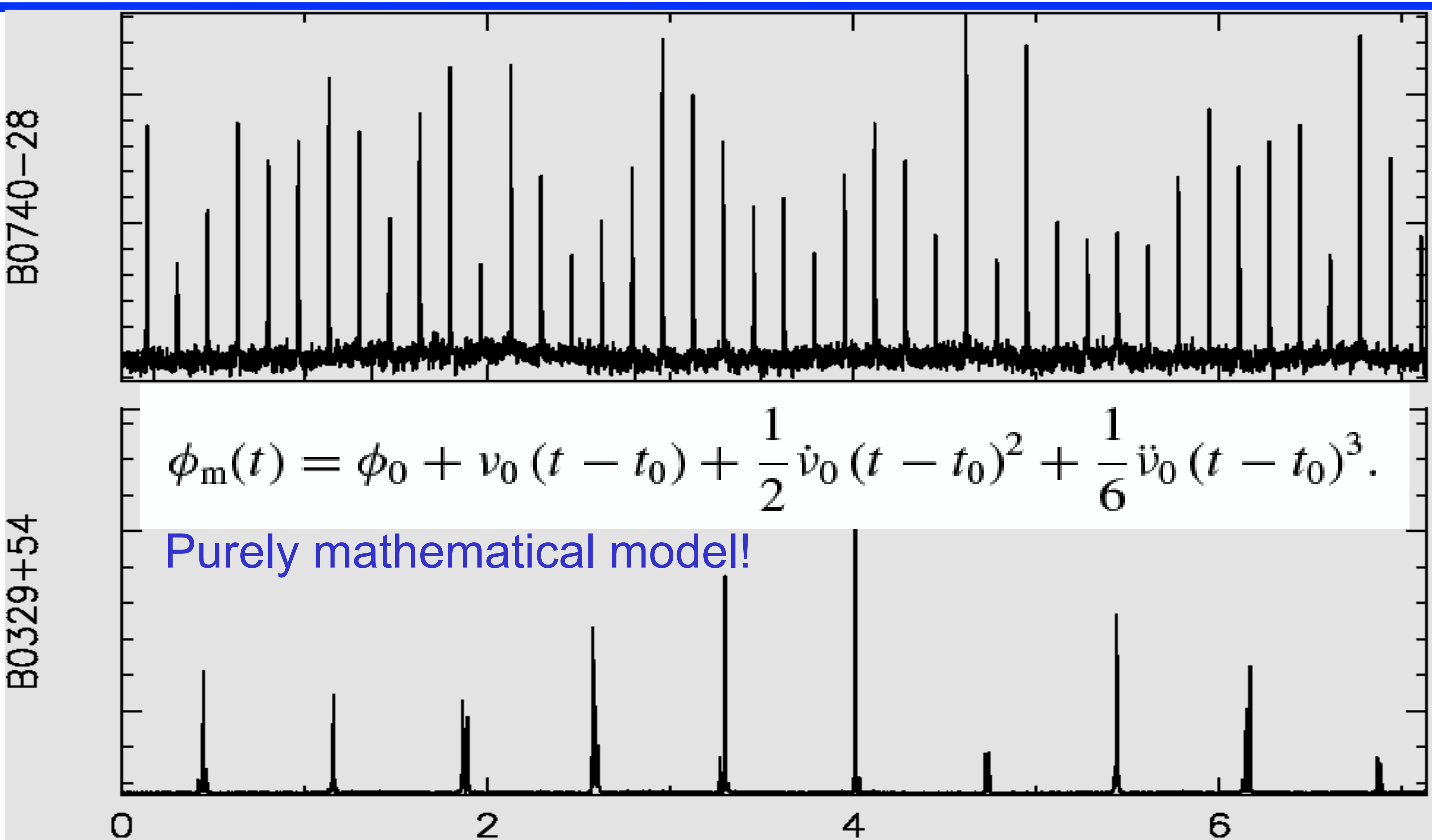
$$n_{\rm b} = \ddot{\nu}\nu/\dot{\nu}^2$$

$$K \equiv \frac{2\mu^2 \sin^2 \alpha}{3Ic^3}$$

$$I\Omega\dot{\Omega} = -\frac{2(BR^3 \sin \chi)^2}{3c^3}\Omega^4$$

Instantaneous spin-down via magnetic dipole radiation,
but *I* and/or *B* and/or *X* may have evolutions or sudden changes.

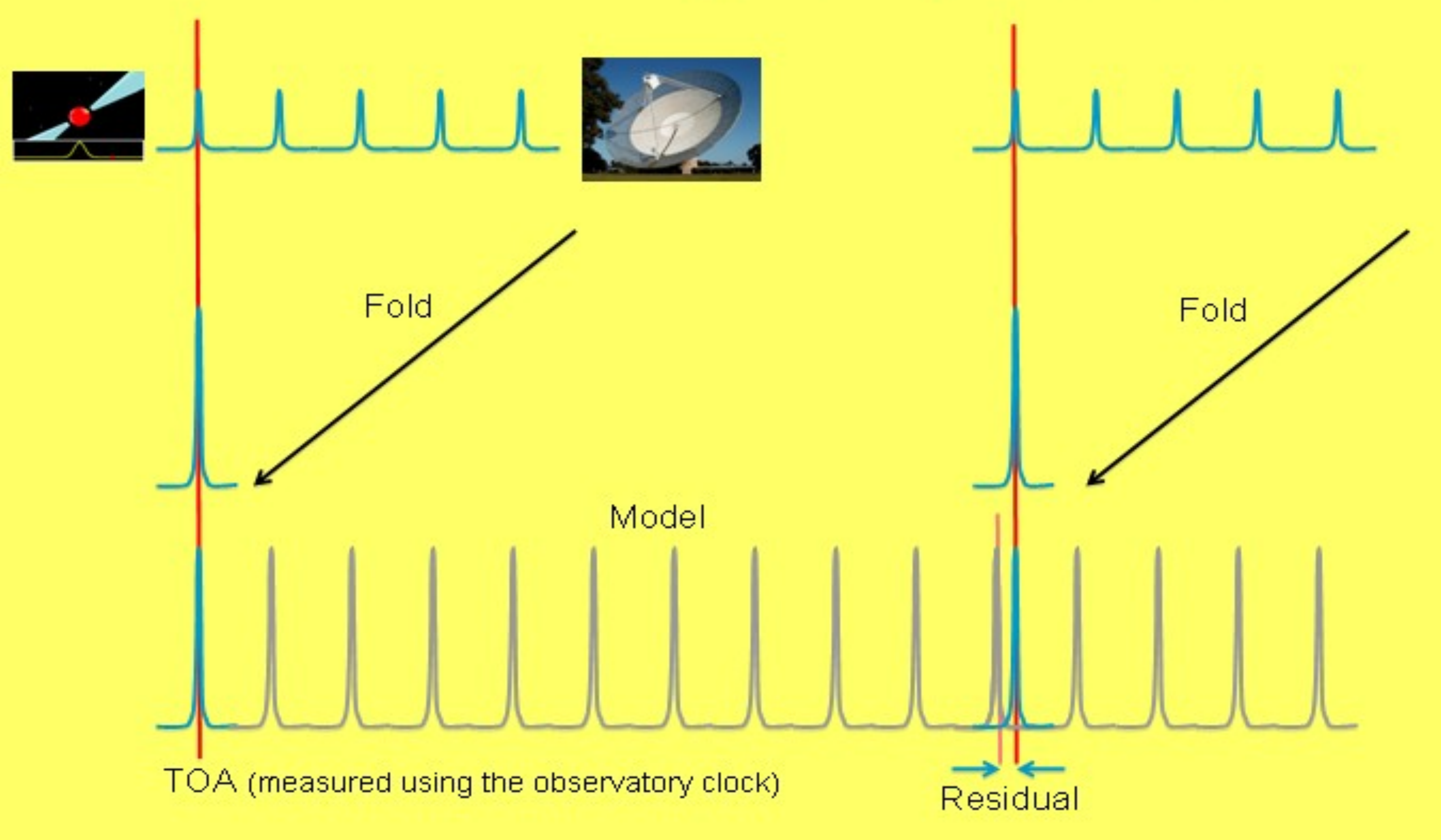
Real observational data of pulsars



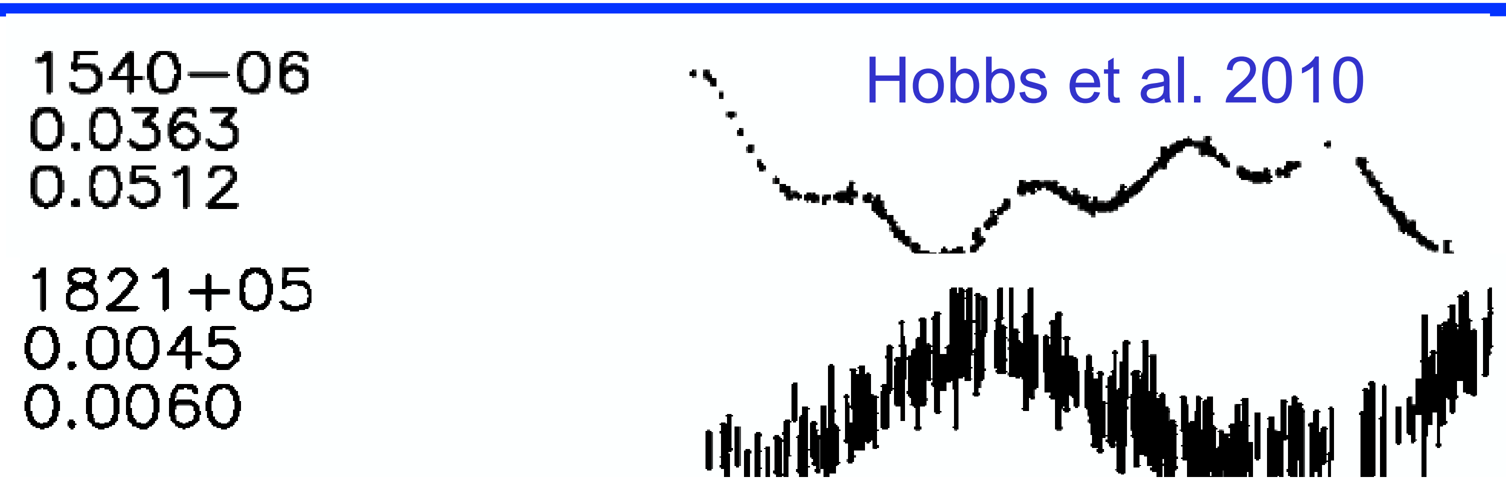
$$\phi_{\rm m}(t) = \phi_0 + \nu_0\,(t - t_0) + \frac{1}{2}\dot{\nu}_0\,(t - t_0)^2 + \frac{1}{6}\ddot{\nu}_0\,(t - t_0)^3.$$

Purely mathematical model!

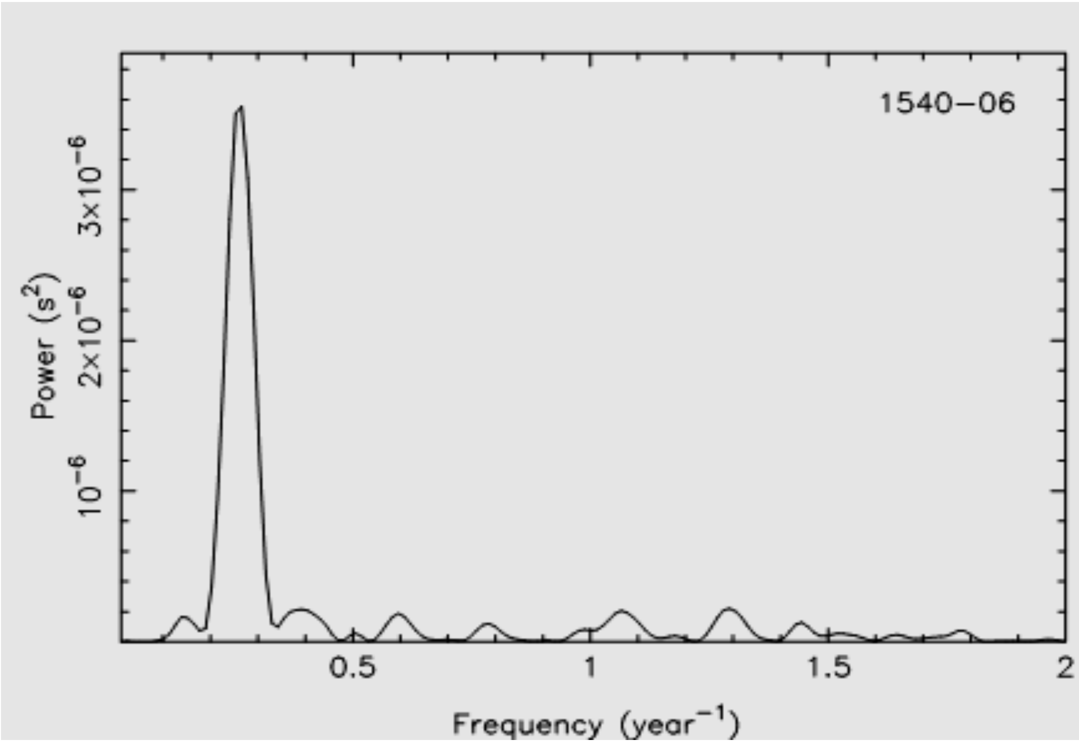
Pulsar timing analysis



Timing Irregularities: (1) timing noise

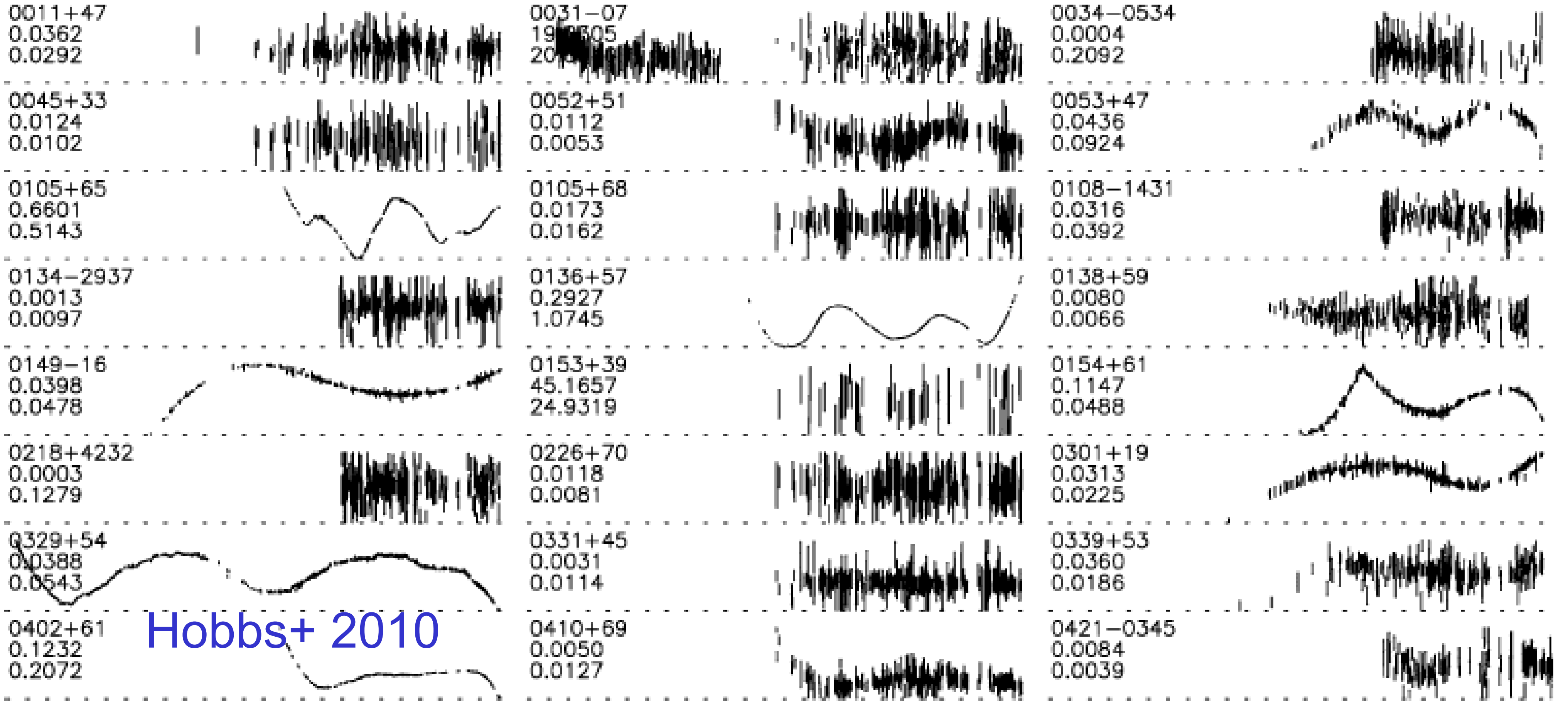


Hobbs et al. 2010



low-frequency structures in timing residuals

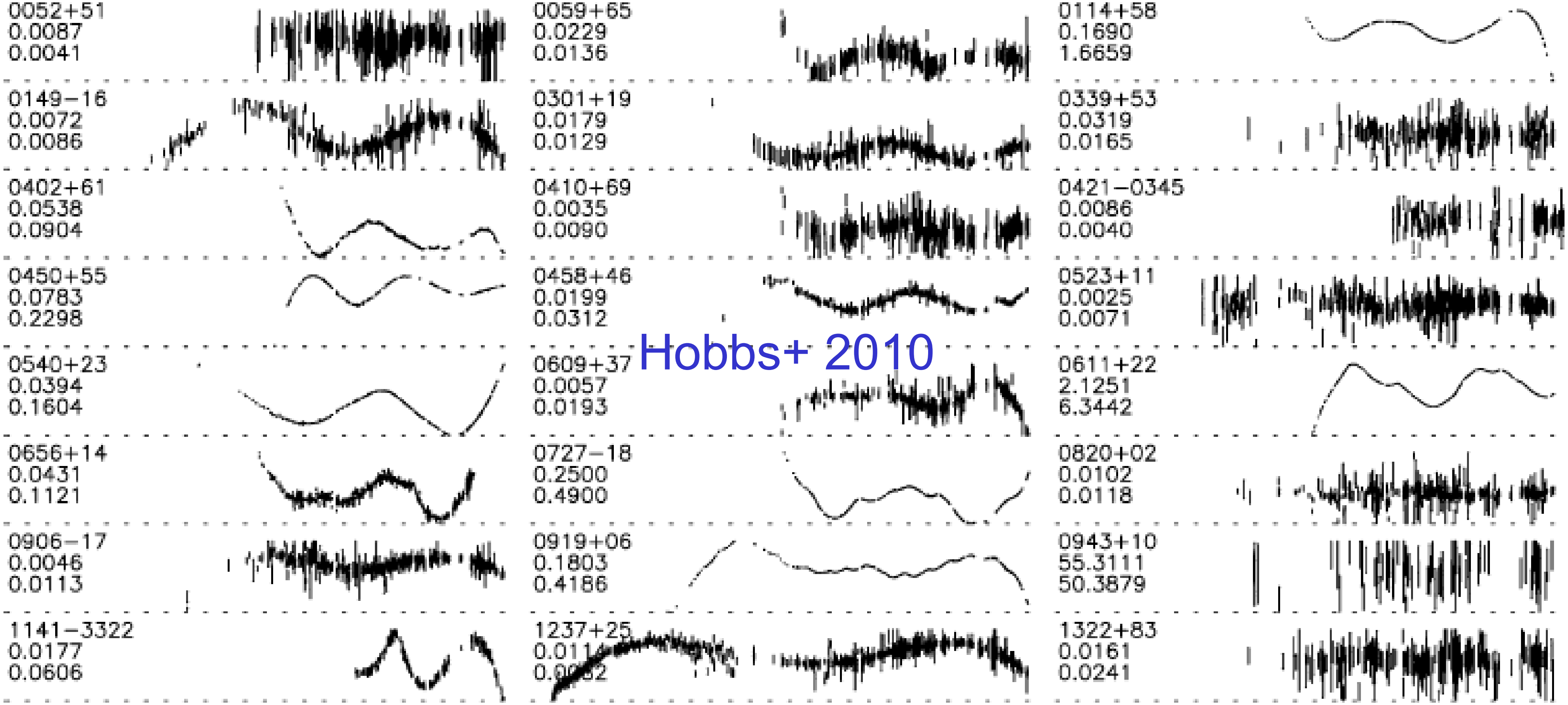
Timing residuals are significant



$$\sigma_1 \geq 10\ \mu\text{s:}\ \ \Phi_i = \Phi_0 + \nu(t_i - t) + \frac{1}{2}\dot{\nu}(t_i - t)^2$$

Significant structures (red noise)

Timing residuals still significant after whitening

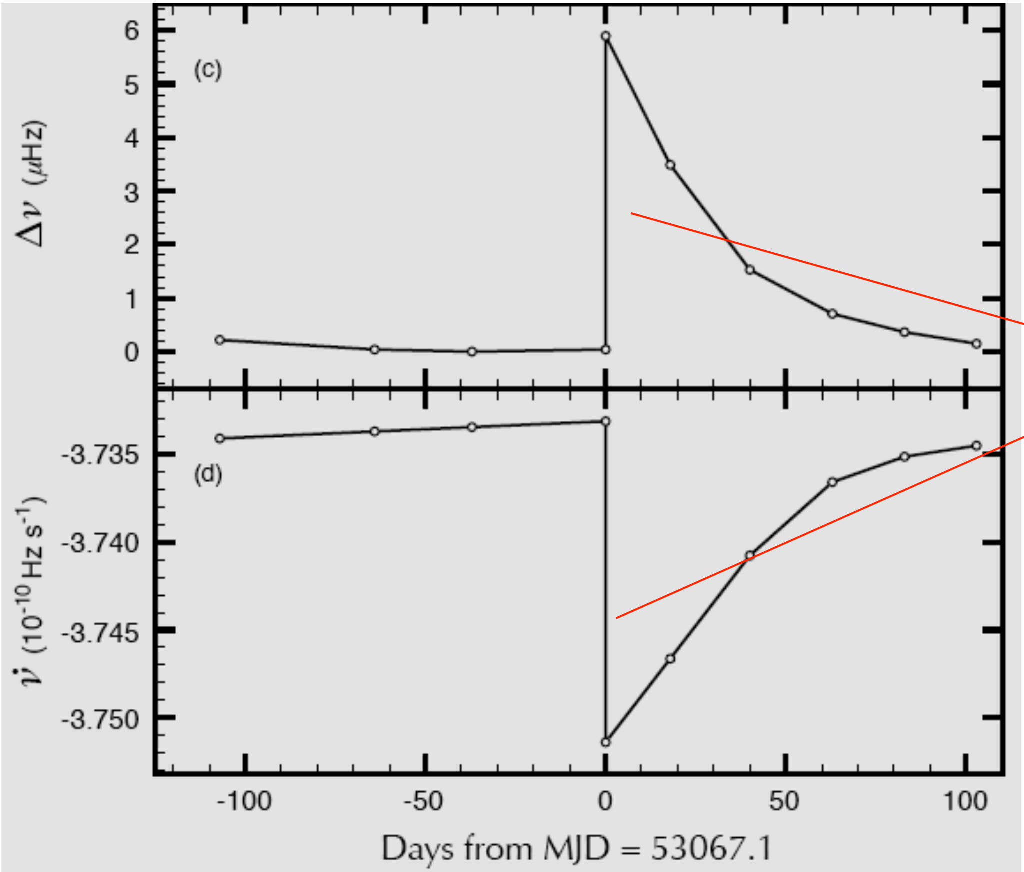


Hobbs+ 2010

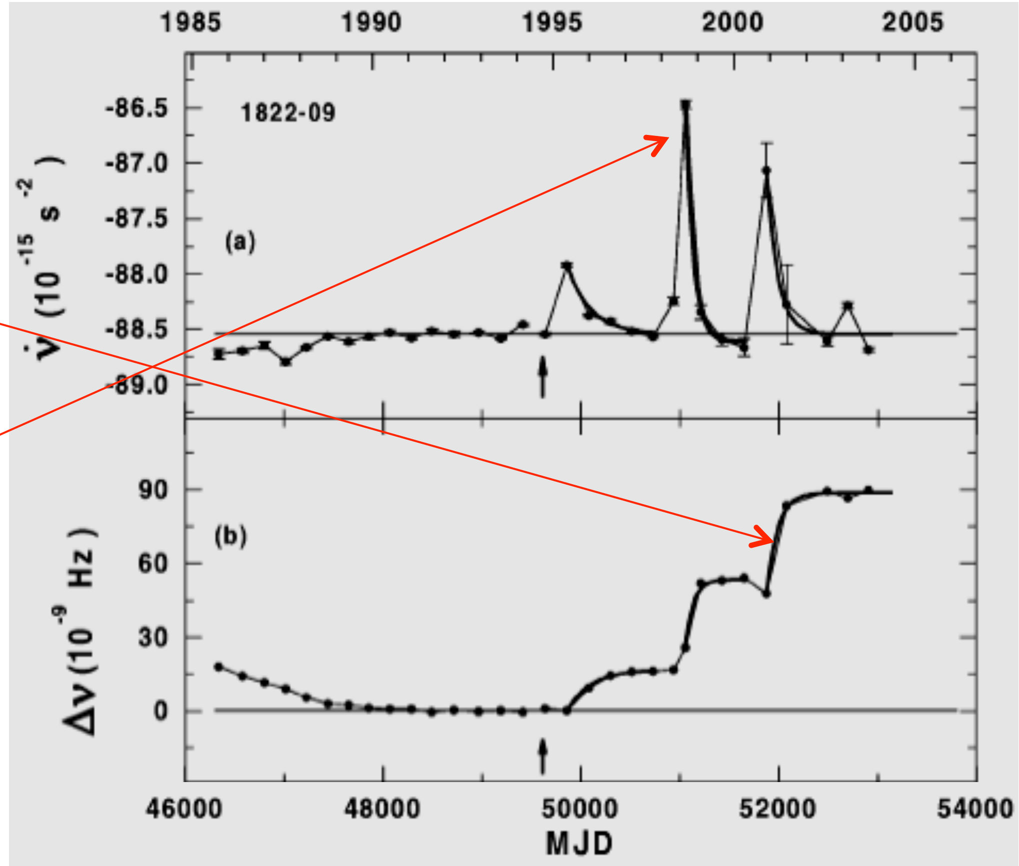
$$\sigma_3 \geq 10\ \mu\text{s:}\ \ \Phi_i = \Phi_0 + \nu(t_i - t) + \frac{1}{2}\dot{\nu}(t_i - t)^2 + \frac{1}{6}\ddot{\nu}_0(t - t_0)^3$$

Still significant structures (red noise)

Timing Irregularities: (2) glitches



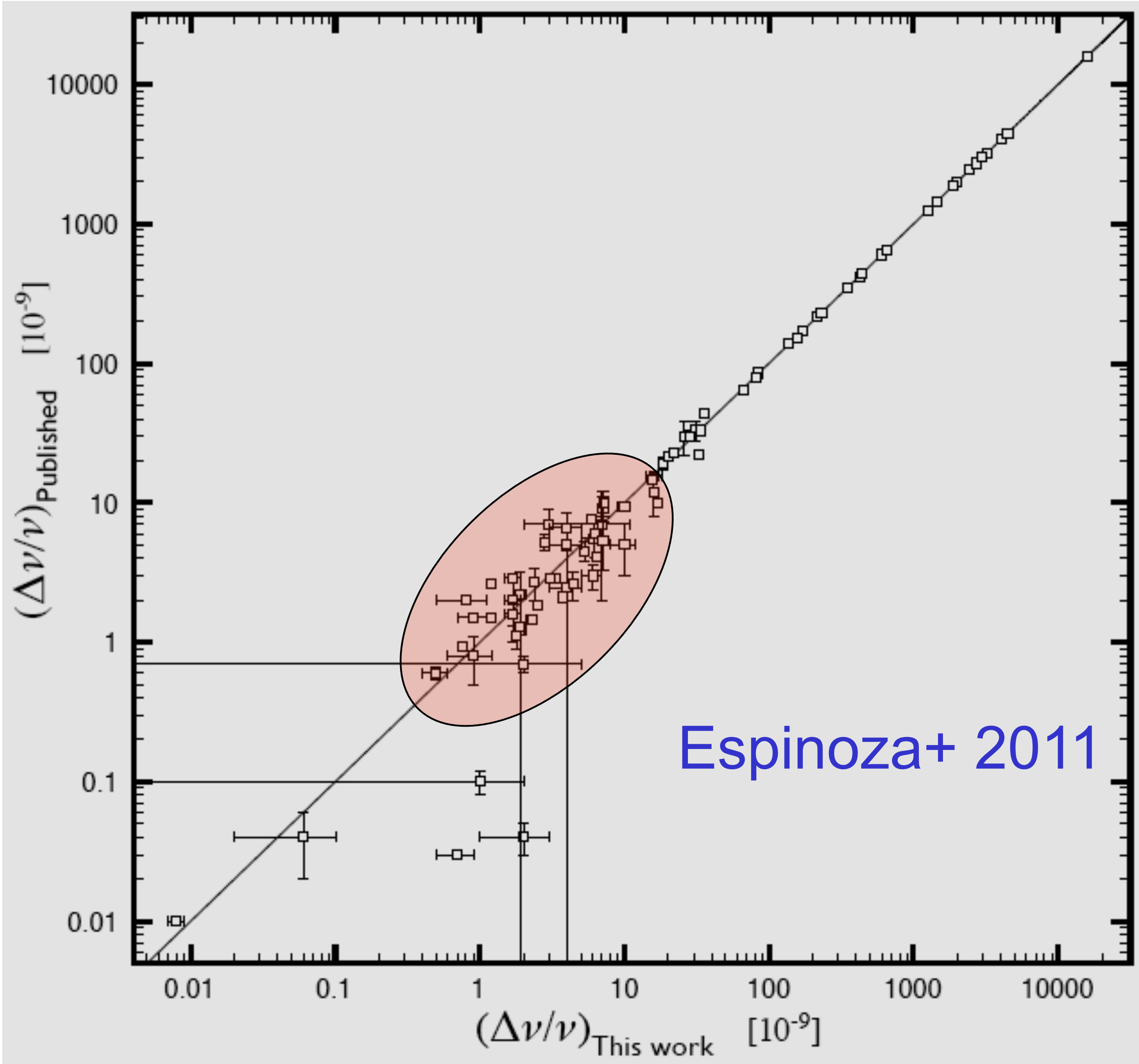
Classical glitch of Crab



Slow glitches of 1822-09

spin and spin-down change suddenly

But different results from different analysis?



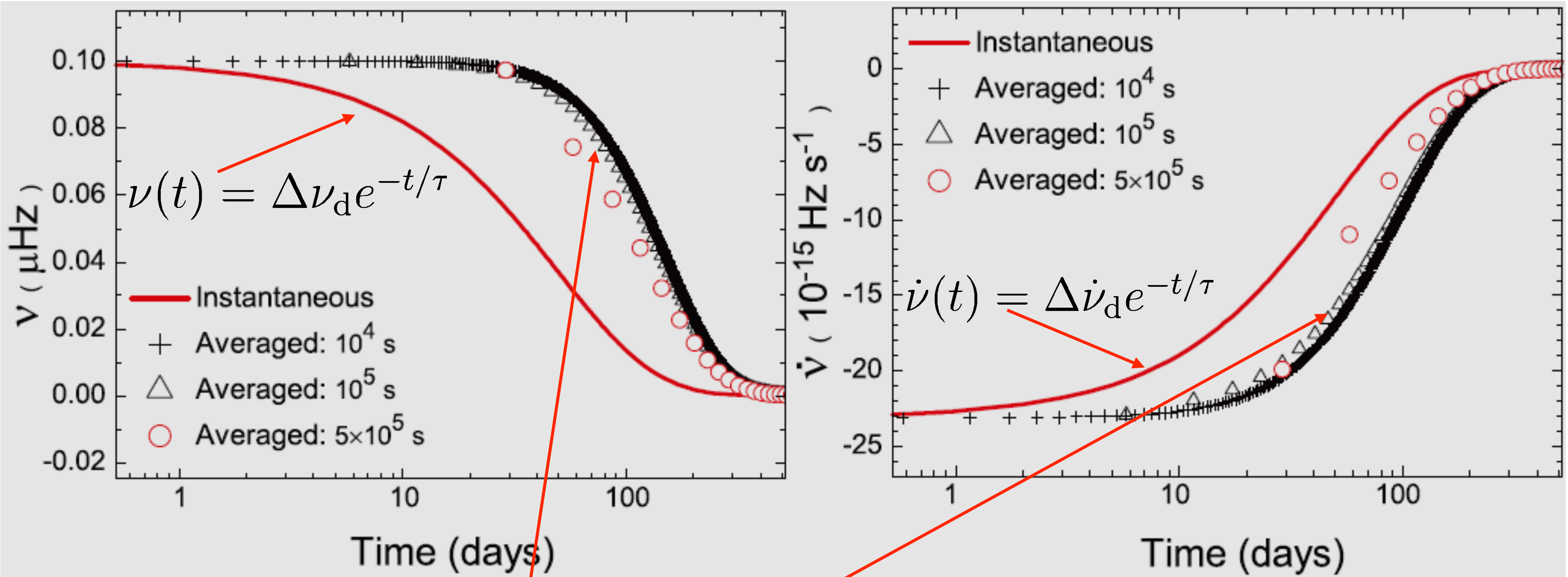
Questions to be addressed in this talk

- How to describe the classical and slow glitches?
- How to describe the timing noise of the spin-down of pulsars beyond the pure mathematical model?
- Limitations on pulsar timing for spacecraft navigation?

1. Observational biases and the relaxation behaviors of slow and classical glitches

Xie, Yi, Zhang, Shuang-Nan, *On the Relaxation Behaviors of Slow and Classical Glitches: Observational Biases and Their Opposite Recovery Trends* , 2013, ApJ778, Issue 1, id. 31, 13pp

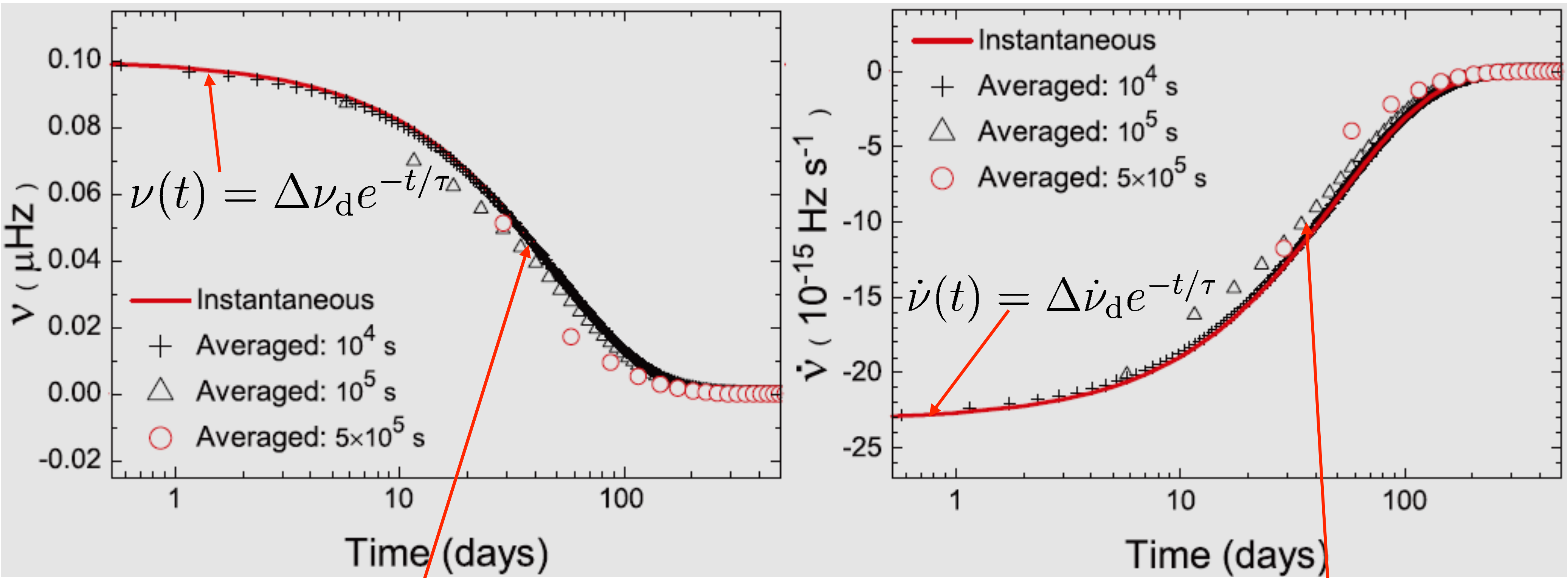
How to get glitch parameters?



$$\Phi_i = \Phi + \nu(t_i - t) + \frac{1}{2}\dot\nu(t_i - t)^2 + \frac{1}{6}\ddot\nu(t_i - t)^3$$
$$i = 1, ..., 10, \ \ (t_{i+1} - t_i) \sim 10^5 \text{ s}$$

Using the "standard" procedure of fitting simulated glitch data, the input model parameters cannot be recovered.

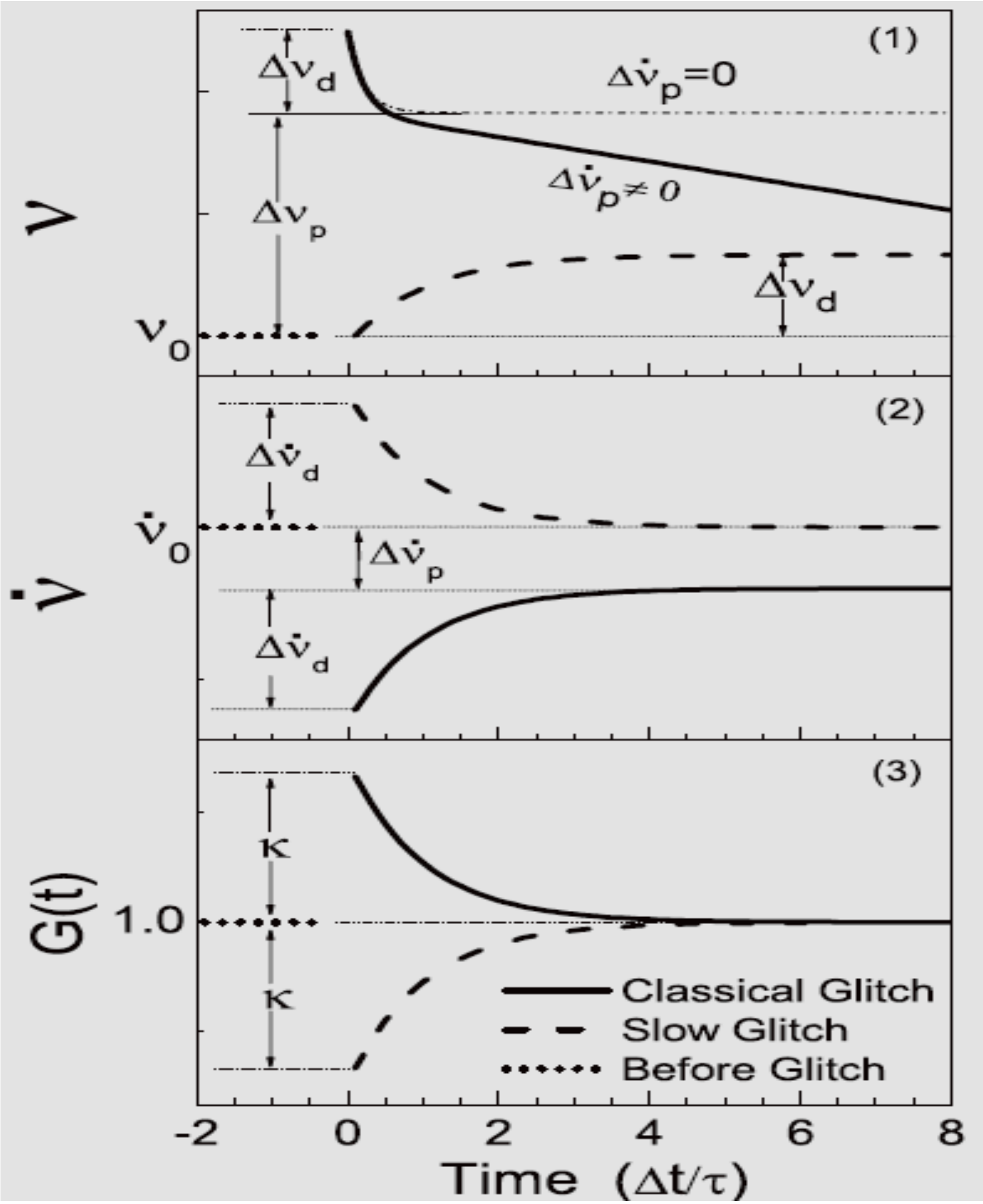
Simple method: TEMPO2



$$\Phi_i = \Phi + \nu(t_i - t) \qquad \Phi_i = \Phi + \nu(t_i - t) + \frac{1}{2}\dot{\nu}(t_i - t)^2$$

Using a simple method of fitting simulated glitch data, the fitted parameters "converge" to the input values when the time bin is shorter than 10^4 s (typically ~ 10^5 s).

Unified description of classical & slow glitches



$$\dot{\Omega}\Omega^{-3} = -\frac{2(BR^3\sin\chi)^2}{3c^3 I}G(t),$$

$$G(t) = 1 + \kappa e^{-\Delta t/\tau},$$

$$\Delta t = t - t_0,$$

$$\dot{\nu}\nu^{-3} = -H_0 G(t),$$

$$H_0 = \frac{8\pi^2(BR^3\sin\chi)^2}{3c^3 I} = 1/2\tau_c\nu_0^2,$$

$$\tau_c = -\nu/2\dot{\nu},$$

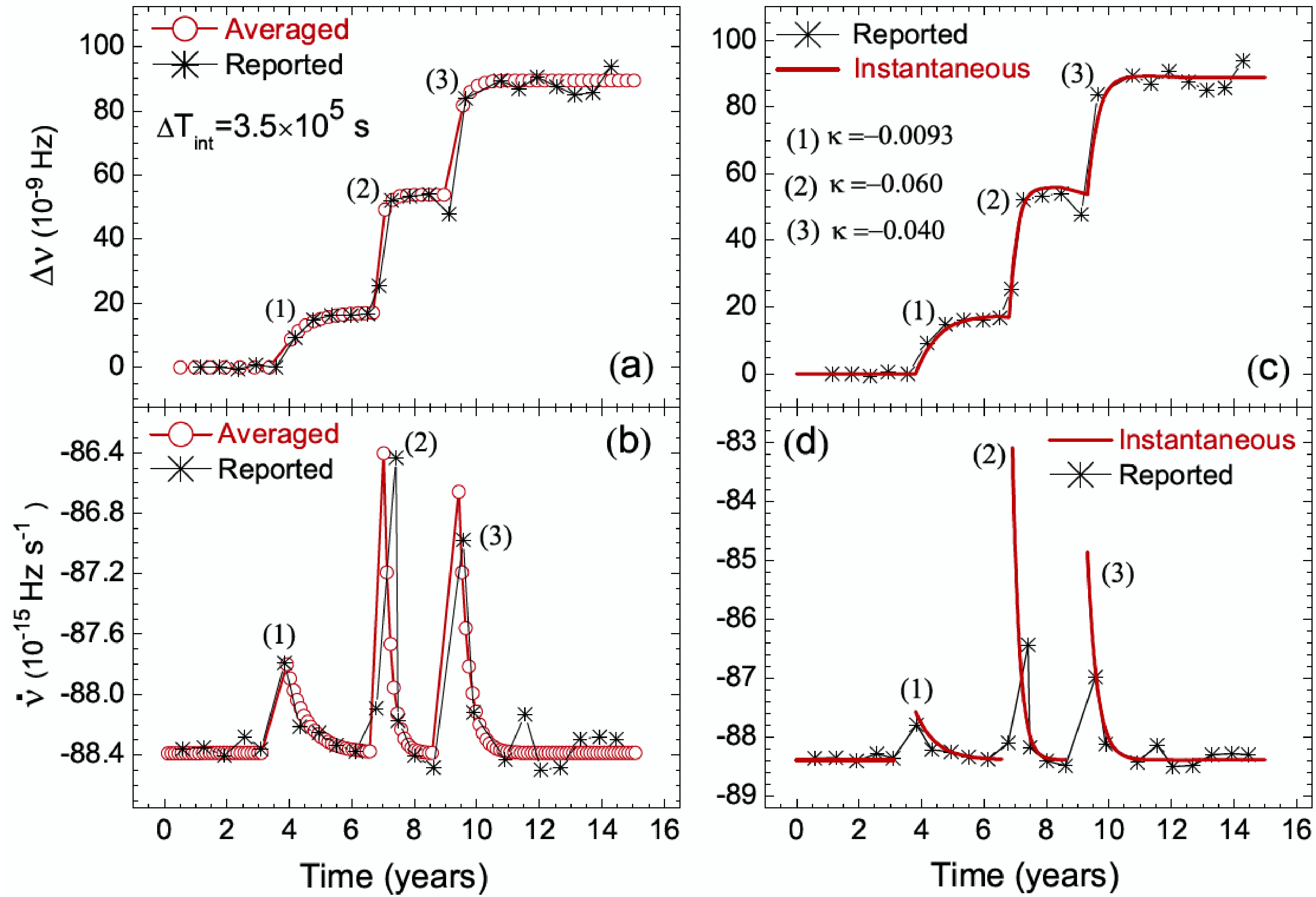
$$\Delta\nu_d = \nu_0\kappa\tau/2\tau_c,$$

$$\Delta\dot{\nu}_d = -\nu_0\kappa/2\tau_c,$$

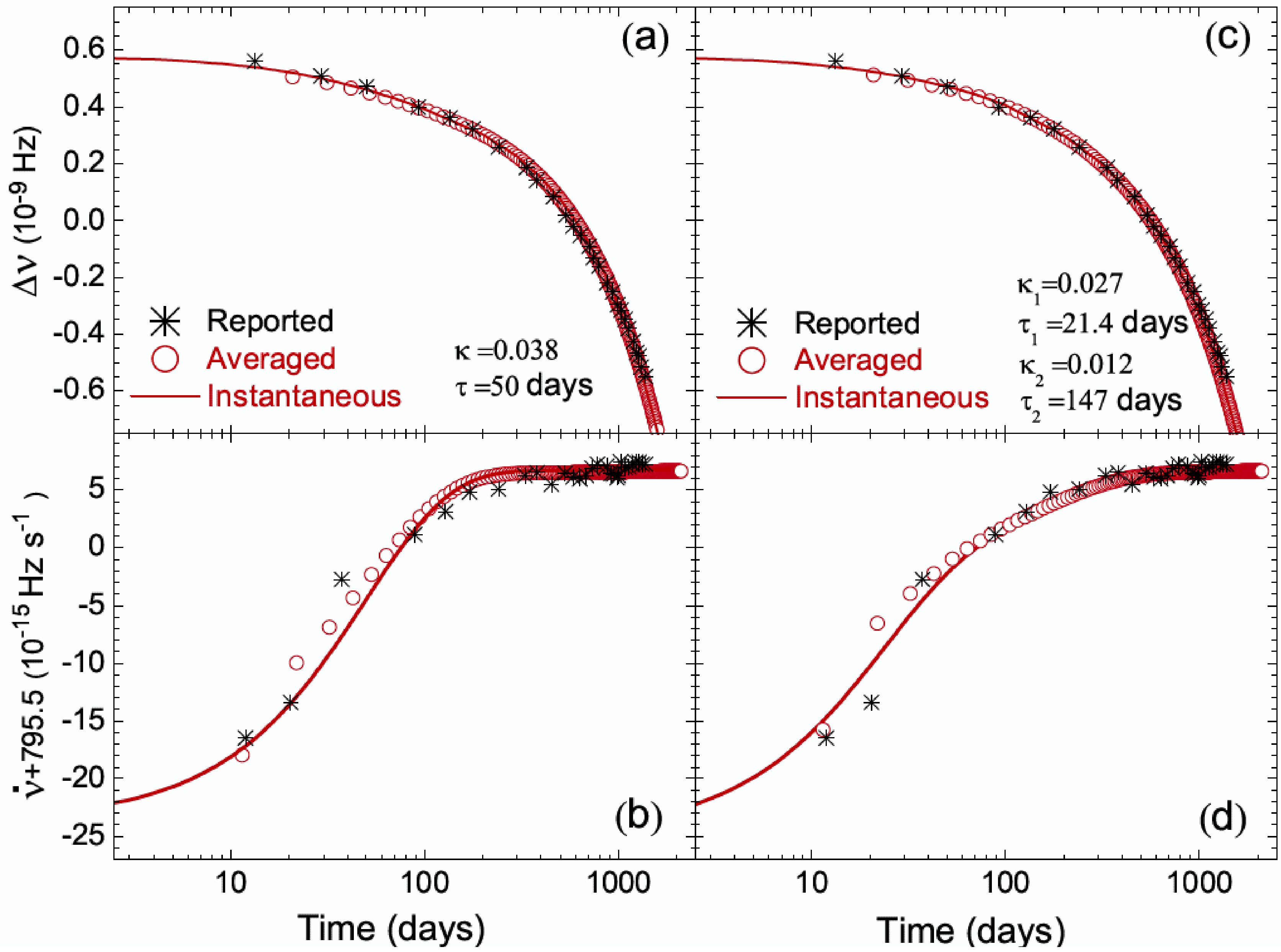
$$\nu(t) \approx \Delta\nu_d e^{-\Delta t/\tau},$$

$$\dot{\nu}(t) \approx \Delta\dot{\nu}_d e^{-\Delta t/\tau}$$

Modeling several slow glitches of B1822-09



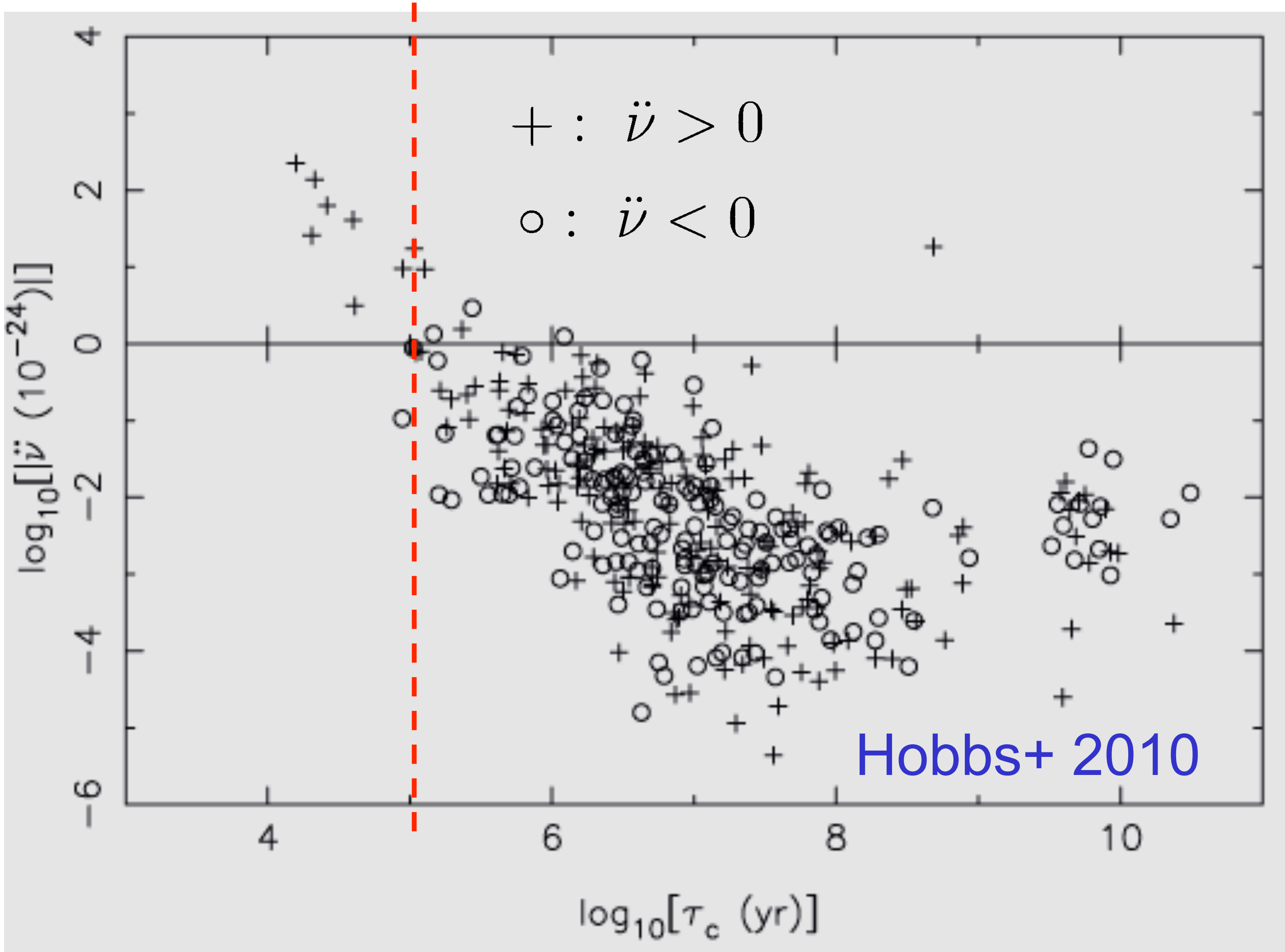
Modeling one classical glitch of B2334+61



2. Phenomenological model of spin and magnetic field evolution of radio pulsars

Zhang, Shuang-Nan, Xie, Yi, *Testing Models of Magnetic Field Evolution of Neutron Stars with the Statistical Properties of Their Spin Evolutions*, 2012, ApJ, 757, 153-160

Statistical properties of pulsar timing noise



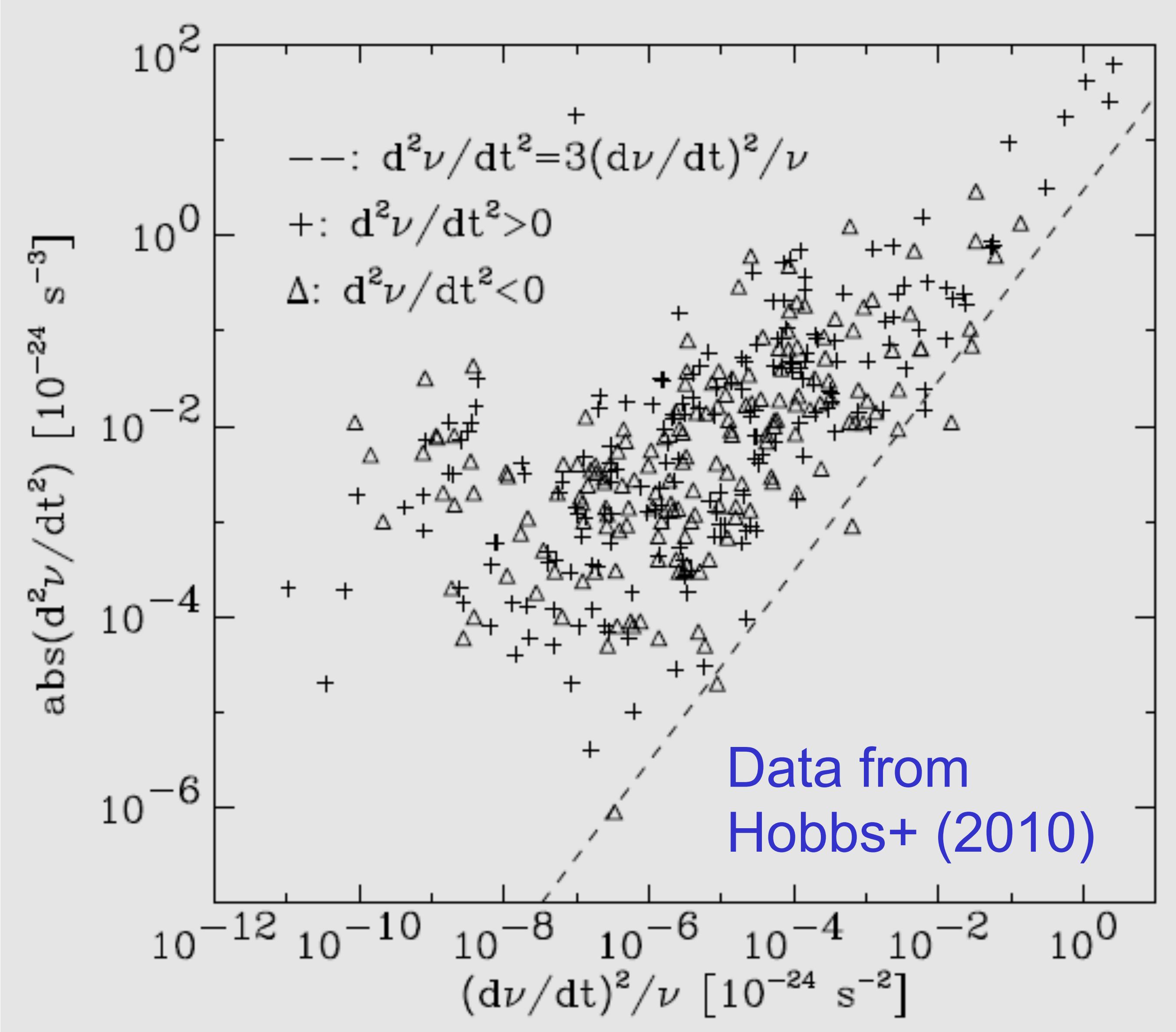
$\ddot{\nu} > 0$ for young pulsars; $\ddot{\nu} > 0$ or < 0 for old pulsars.

Testing the standard magnetic dipole radiation model

$$\Omega\dot{\Omega} = -\frac{(BR^3)^2}{6c^3}\Omega^4$$
$$\dot{\nu} = -AB^2\nu^3,$$
$$A = \frac{(2\pi R^3)^2}{6c^3 I}$$

$\dot{B} = 0 \Rightarrow$
$\ddot{\nu} = 3\dot{\nu}^2/\nu > 0$
But data $\Rightarrow$
$\ddot{\nu} \gg 3\dot{\nu}^2/\nu$
Model fails!

$\dot{B} \neq 0 \Rightarrow$
$\ddot{\nu} = 3\dot{\nu}^2/\nu + 2\dot{\nu}\dot{B}/B$
$\ddot{\nu} \approx 2\dot{\nu}\dot{B}/B \Rightarrow$
$\dot{B} > 0$ or < 0



Evidence for evolution of B

Define $\Omega\dot{\Omega} = -K\Omega^4$

$\Rightarrow$

$K = AB^2$

$\dot{K} = (3\dot{\nu}^2 - \ddot{\nu}\nu)/\nu^4$

Define

$\tau_K = K/|\dot{K}| = B/2|\dot{B}|$

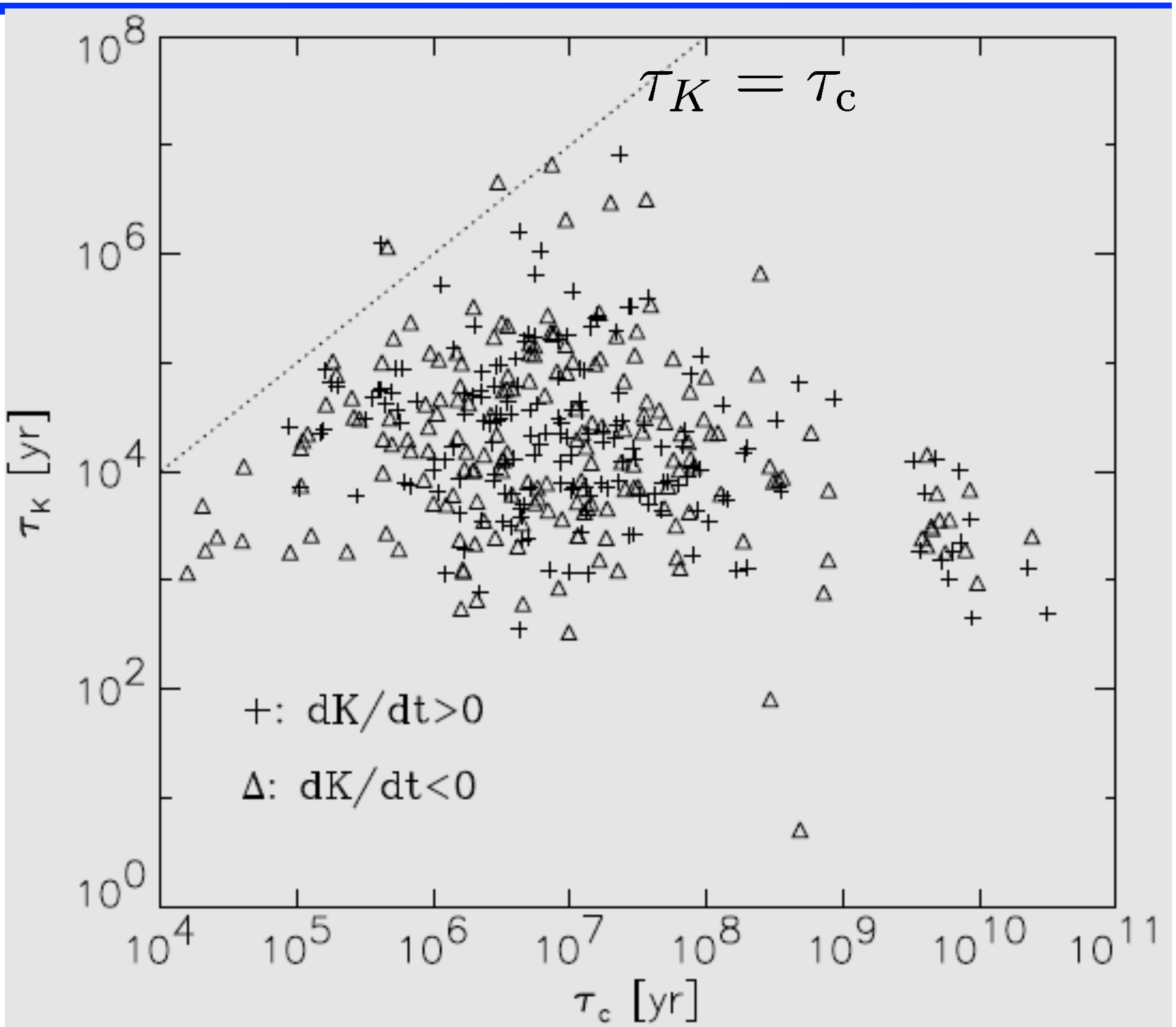
$\Rightarrow$

$\tau_K = |(\dot{\nu}\nu)/(3\dot{\nu}^2 - \ddot{\nu}\nu)|$

Data$\Rightarrow$ $\tau_K \ll \tau_c$

$\Rightarrow$

Significant B evolution!



Effective short term *B*-oscillation

Beam width

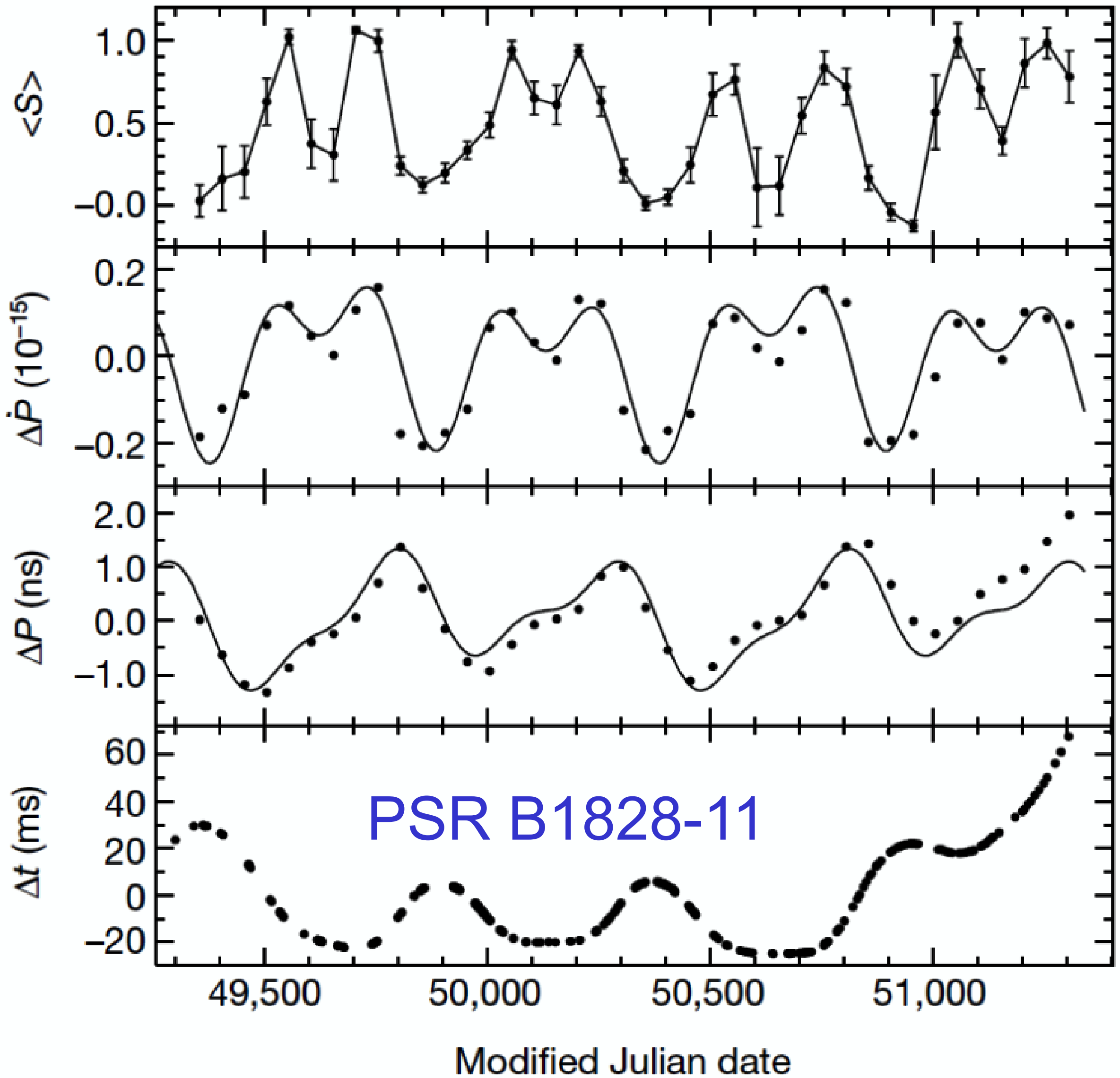
$$\phi_{fit}(t) = (\phi_0 + \nu t + \frac{1}{2}\dot{\nu} t^2)$$

$$\dot{P} = 8\pi^2 m^2 \sin^2\alpha/(3c^3 IP)$$

$$\alpha = 60^\circ + 0.3\sin(\phi_0 + 2\pi t/T)$$

Stairs et al. 2000, Nature

$$B^2 \sin^2\alpha = \frac{3c^3 I}{8\pi^2 r_*^6} P\dot{P}$$



Analytical model of pulsar spin with *B*-evolution

Assume $B(t) = B_{\rm d}(t)(1 + k\sin(\phi + 2\pi\frac{t}{T}))$
$T \ll B_{\rm d}/\dot{B}_{\rm d}$ & $f = 2\pi k/T \Rightarrow$
$\dot{B} \simeq \dot{B}_{\rm d} + B_{\rm d} f\cos(\phi + 2\pi\frac{t}{T}) \Rightarrow \dot{B} \simeq \dot{B}_{\rm d} \pm f B_{\rm d}$

For exponential decay $B_{\rm d} = B_0\exp(-t/\tau_{\rm d})$
$\ddot{\nu} \approx 2\dot{\nu}\dot{B}/B = -2\dot{\nu}(1/\tau_{\rm d} \pm f)$
Cannot explain why $\ddot{\nu} > 0$ for young pulsars,
but $\ddot{\nu} > 0$ or < 0 for old pulsars.
$\Rightarrow$ exponential decay model rejected!

For power-law decay $B_{\rm d} = B_0(t/t_0)^{-\alpha}$
$\ddot{\nu} \approx 2\dot{\nu}\dot{B}/B = -2\dot{\nu}(\alpha/t \pm f) \Rightarrow$
For young pulsars $\alpha/t > f \Rightarrow \ddot{\nu} > 0$;
For old pulsars $\alpha/t < f \Rightarrow \ddot{\nu} > 0$ or < 0

Testing the Model of Power-law Decay Modulated by Oscillations with All Pulsars

$$\ddot{\nu} \simeq \eta(-\dot{\nu})^{1+\beta}/\nu^{3\beta} \pm 2\dot{\nu} f,$$

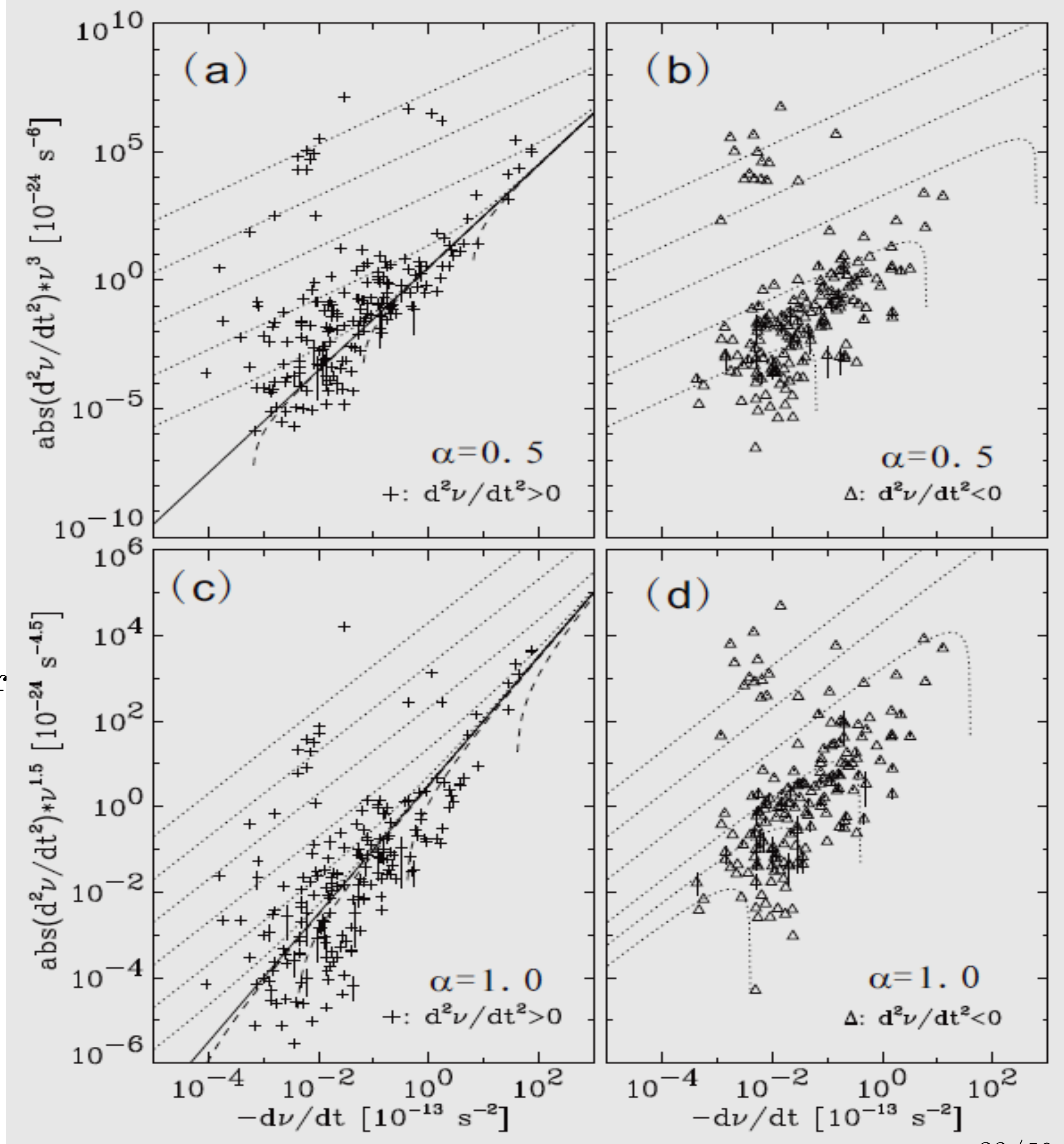
$$\Rightarrow$$

$$\ddot{\nu}\nu^{3\beta} \simeq \eta(-\dot{\nu})^{1+\beta} \pm 2\dot{\nu}\nu^{3\beta} f$$

Solid line: *f*=0

Other lines:

f=10^{-14}-10^{-10}

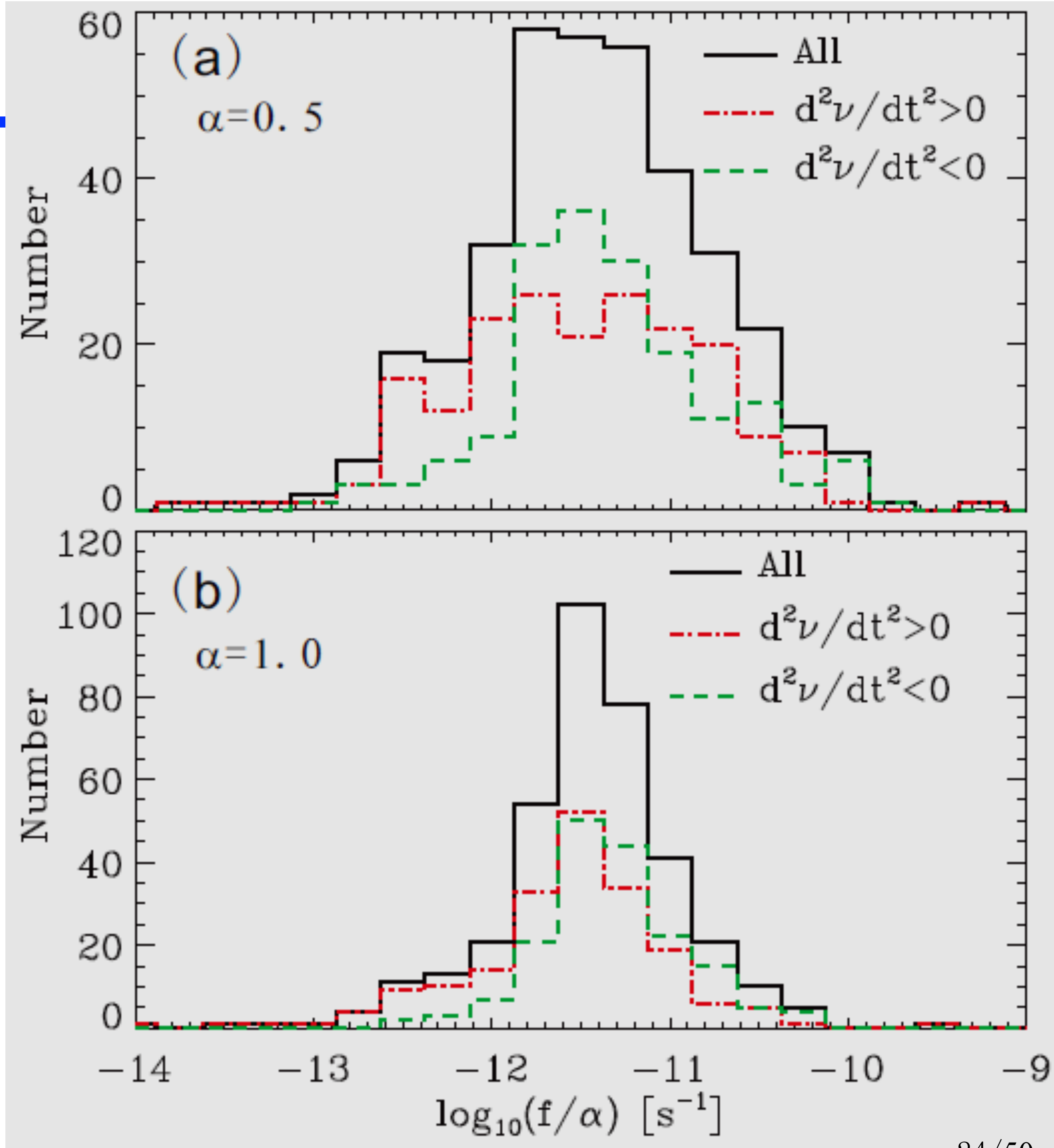


Testing the Model of Power-law Decay Modulated by Oscillations with All Pulsars

$$\ddot{\nu} \simeq \eta(-\dot{\nu})^{1+\beta}/\nu^{3\beta} \pm 2\dot{\nu} f,$$
$$\Rightarrow$$
$$f = |(\ddot{\nu} - \eta(-\dot{\nu})^{1+\beta}/\nu^{3\beta})/2\dot{\nu}|$$

Data⇒

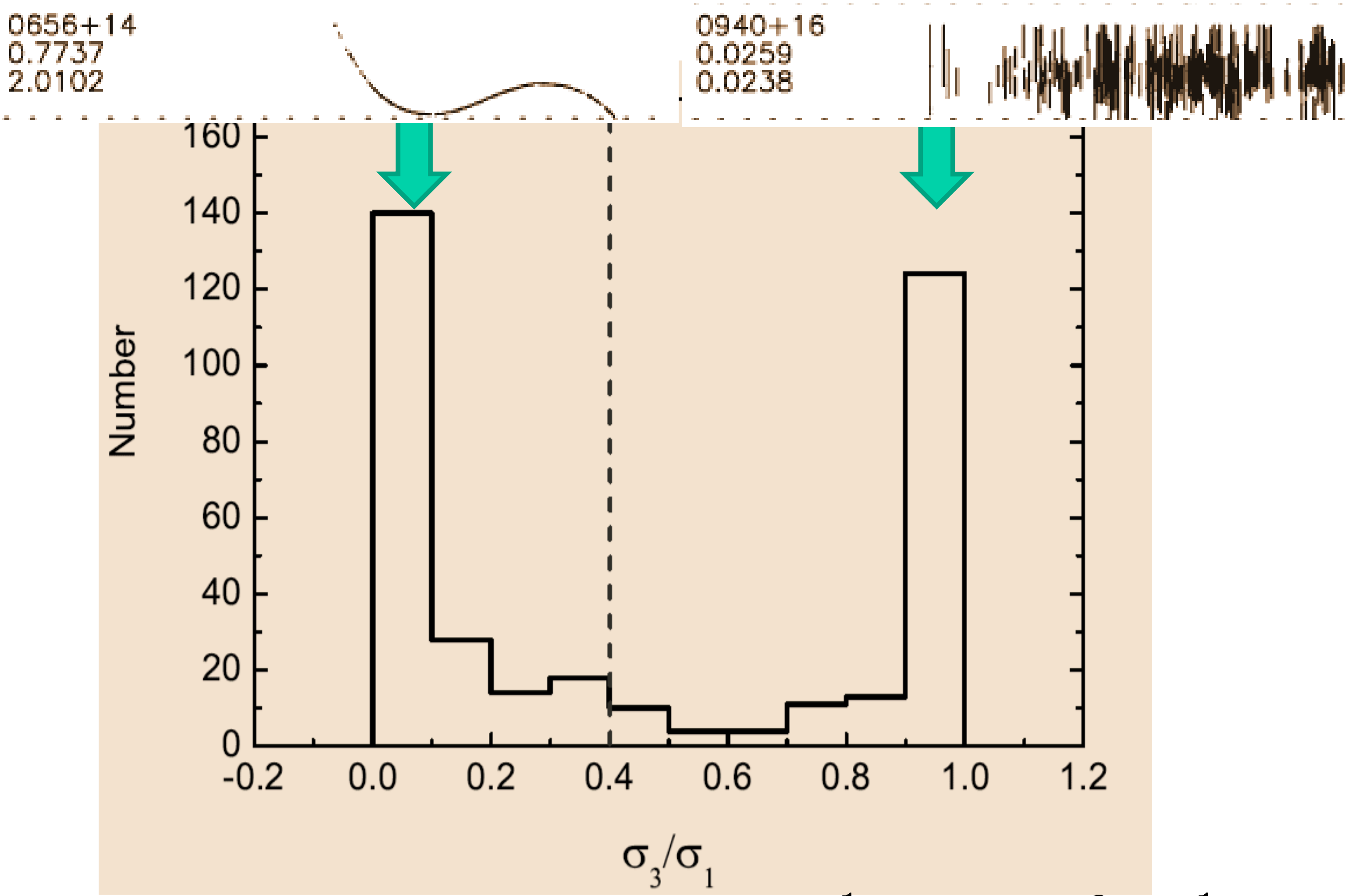
f has a narrow distribution!



3. Understanding the residual patterns of timing solutions of radio pulsars with a model of magnetic field oscillation

Xudong Gao et al 2015, submitted.

Sample selection: significant residual patterns

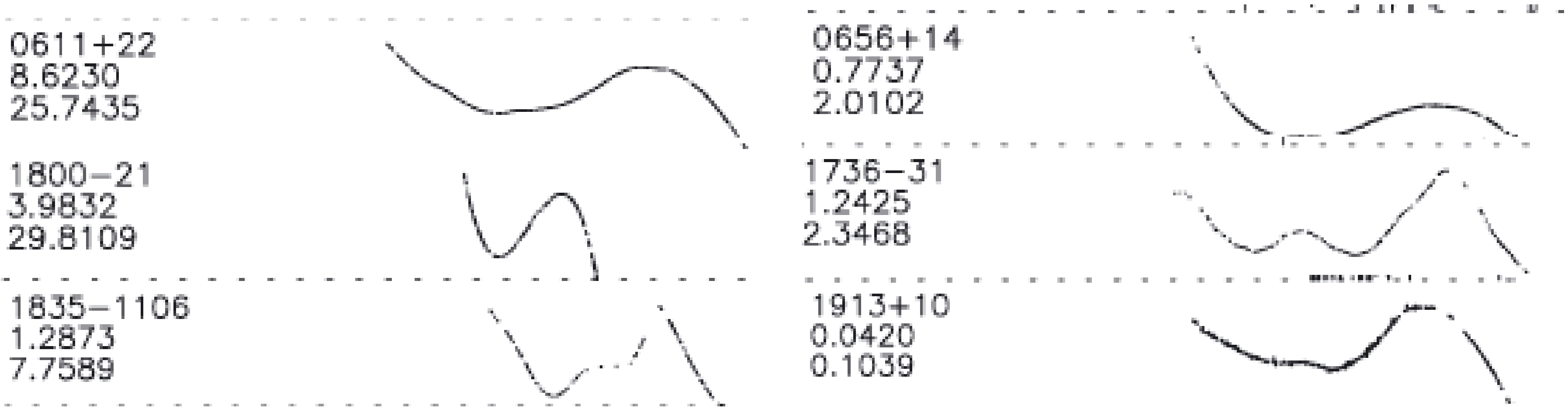


$$\sigma_3 : \text{residuals} : \Phi_i = \Phi_0 + \nu(t_i - t) + \frac{1}{2}\dot{\nu}(t_i - t)^2 + \frac{1}{6}\ddot{\nu}(t_i - t)^3$$

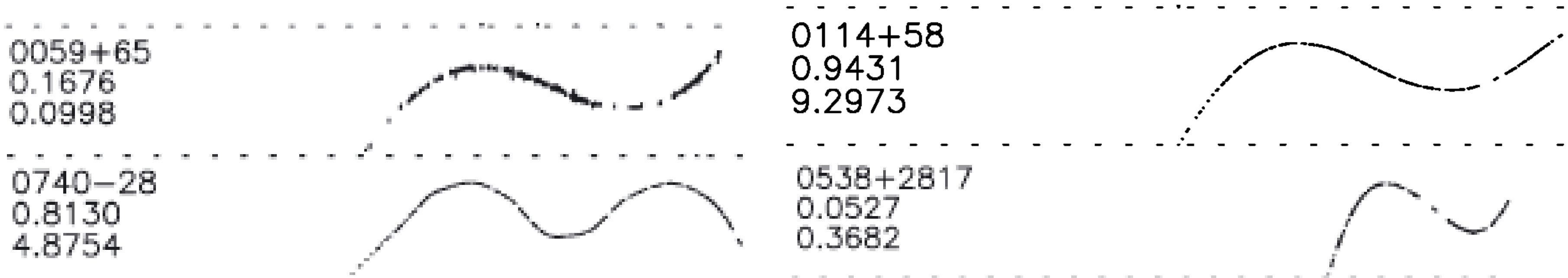
$$\sigma_1 : \text{residuals} : \Phi_i = \Phi_0 + \nu(t_i - t) + \frac{1}{2}\dot{\nu}(t_i - t)^2$$

Two classes

$\ddot{\nu} > 0$: M mode



$\ddot{\nu} < 0$: W mode



Detailed classification

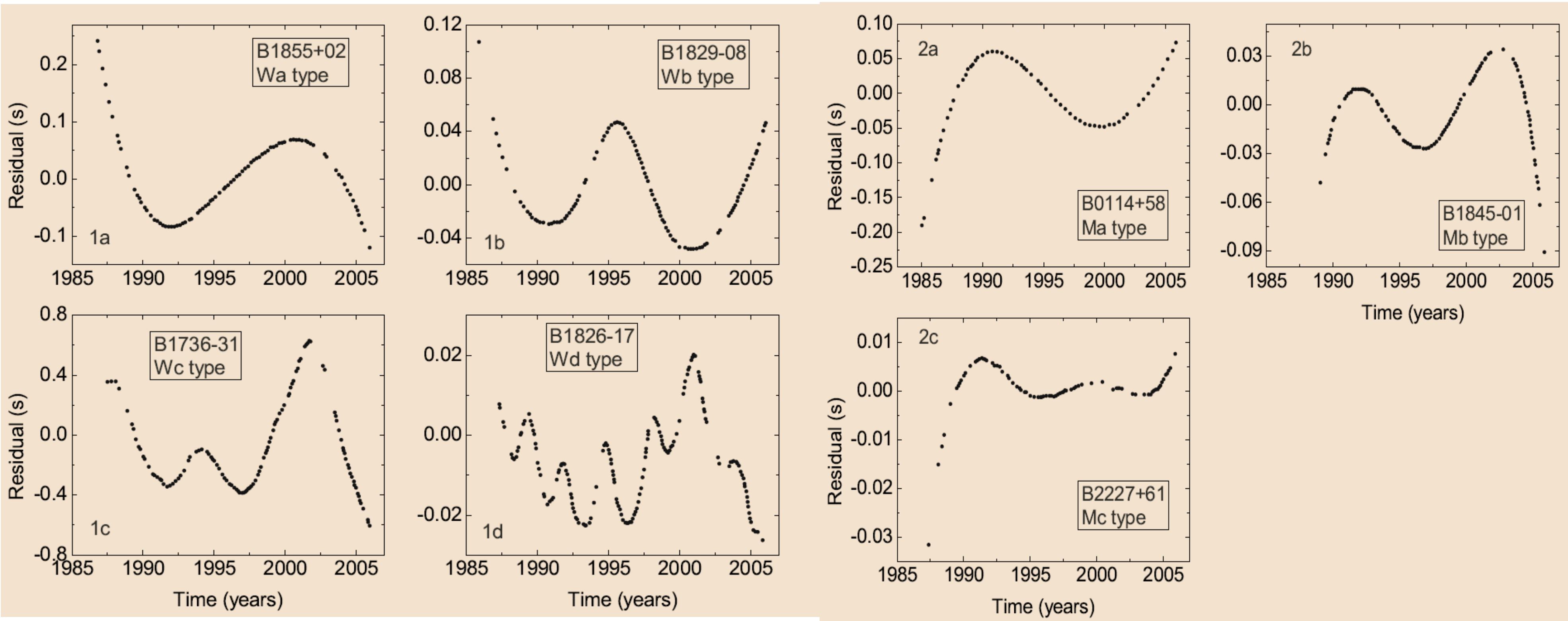
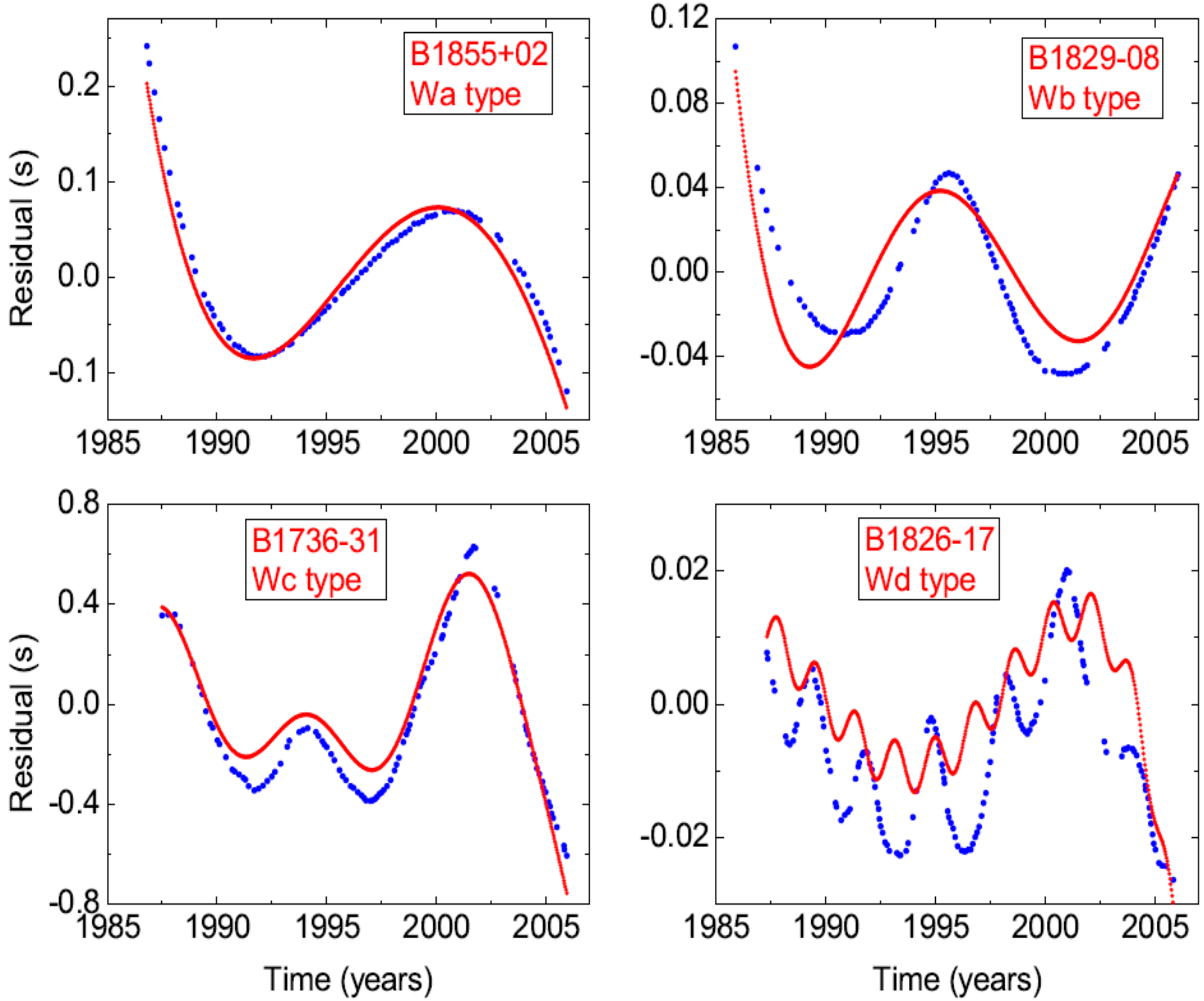


Table 1: Number of the corresponding types in our sample

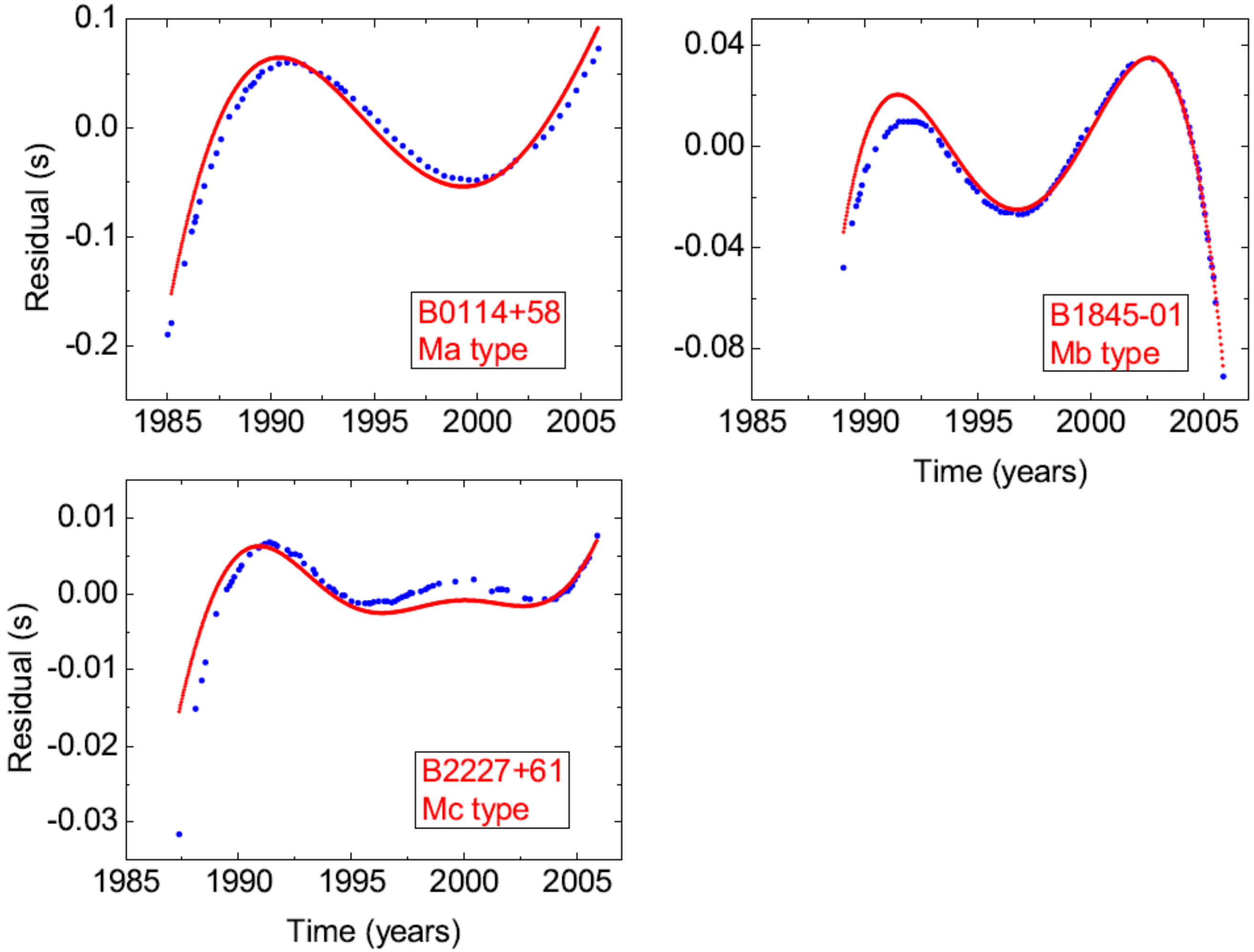
Type	*Wa*	*Wb*	*Wc*	*Wd*
Number	60	13	22	6
Type	*Ma*	*Mb*	*Mc*	
Number	63	19	7	

Fitting the residuals with our model



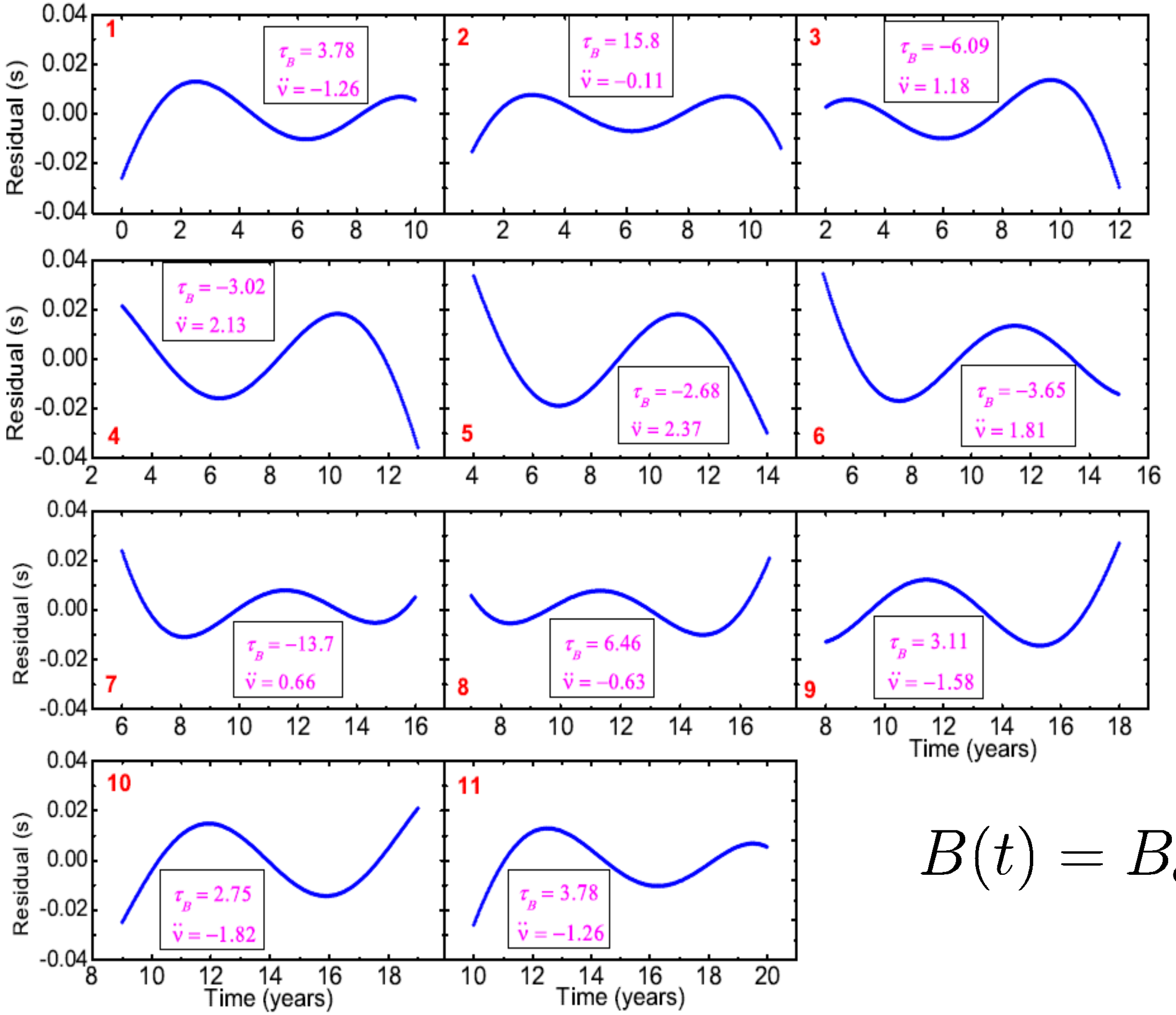
$$B(t) = B_{\rm d}(t)(1 + k\sin(\phi + 2\pi \frac{t}{T})),\ B_{\rm d} = B_0(t/t_0)^{-\alpha}$$

Fitting the residuals with our model



$$B(t) = B_{\rm d}(t)(1 + k\sin(\phi + 2\pi\frac{t}{T})),\ B_{\rm d} = B_0(t/t_0)^{-\alpha}$$

Simulate the pattern evolution in a single pulsar



All patterns are reproduced with different observation time spans: mostly Ma and Wa patterns.

These patterns can be understood with a simple model!

$$B(t) = B_{\rm d}(t)(1 + k\sin(\phi + 2\pi\frac{t}{T}))$$

4. Why do the observed braking indices of pulsars span a range of more than 100 millions?

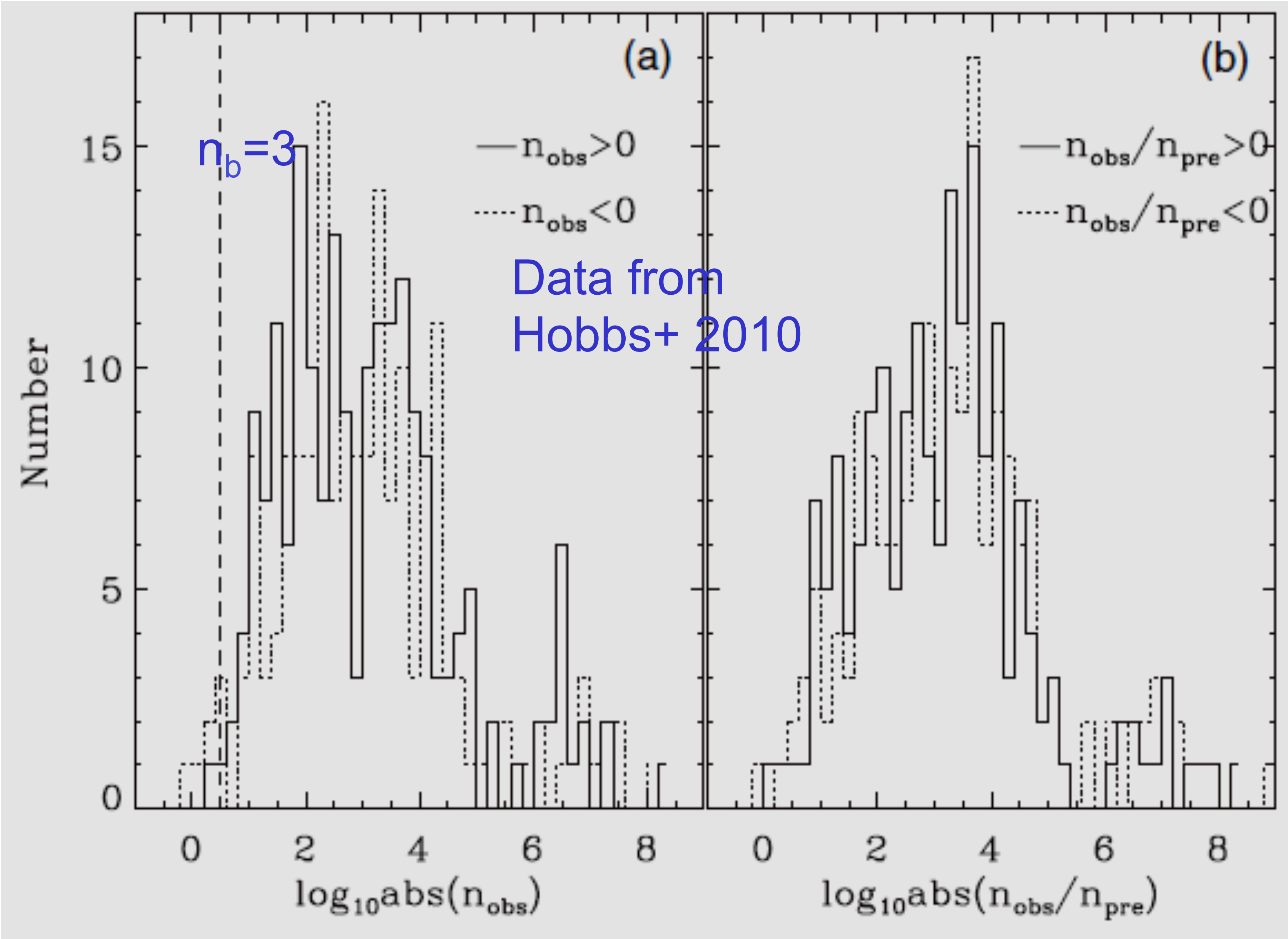
Zhang, Shuang-Nan, Xie, Yi, *Why Do the Braking Indices of Pulsars Span a Range of More Than 100 Millions?* ApJ, 761, 102 (2012).

Braking law and braking index (1)

Taking the braking law as $\dot{\Omega} = -K\Omega^{n_\mathrm{b}}$, Manchester & Taylor (1977) found the braking index $n_\mathrm{b} = \ddot{\nu}\nu/\dot{\nu}^2$, if $\dot{K} = 0$.
$n_\mathrm{b} = 3$ in the standard B-dipole radiation model.

Magalhaes+ (2012) found $n_\mathrm{b} = \frac{\log(|\dot{\Omega}|S^2/\xi^2)}{\log(\Omega)}$, where $\Omega = 2\pi\nu$,
$S \equiv \sqrt{\frac{3c^3}{2}\frac{\lambda}{\sin^2\alpha}\frac{M}{R^4}} = 2.3 \times 10^{20}\ \mathrm{Hz}^{1/2}\ \mathrm{G}$ and is assumed to be the same for all pulsars, $\xi = \pm S\sqrt{\frac{|\dot{\Omega}|}{\Omega^{n_\mathrm{b}}}}$ in units of $\mathrm{Hz}^{(3-n_\mathrm{b})/2}$ G.

Observed braking indices of pulsars

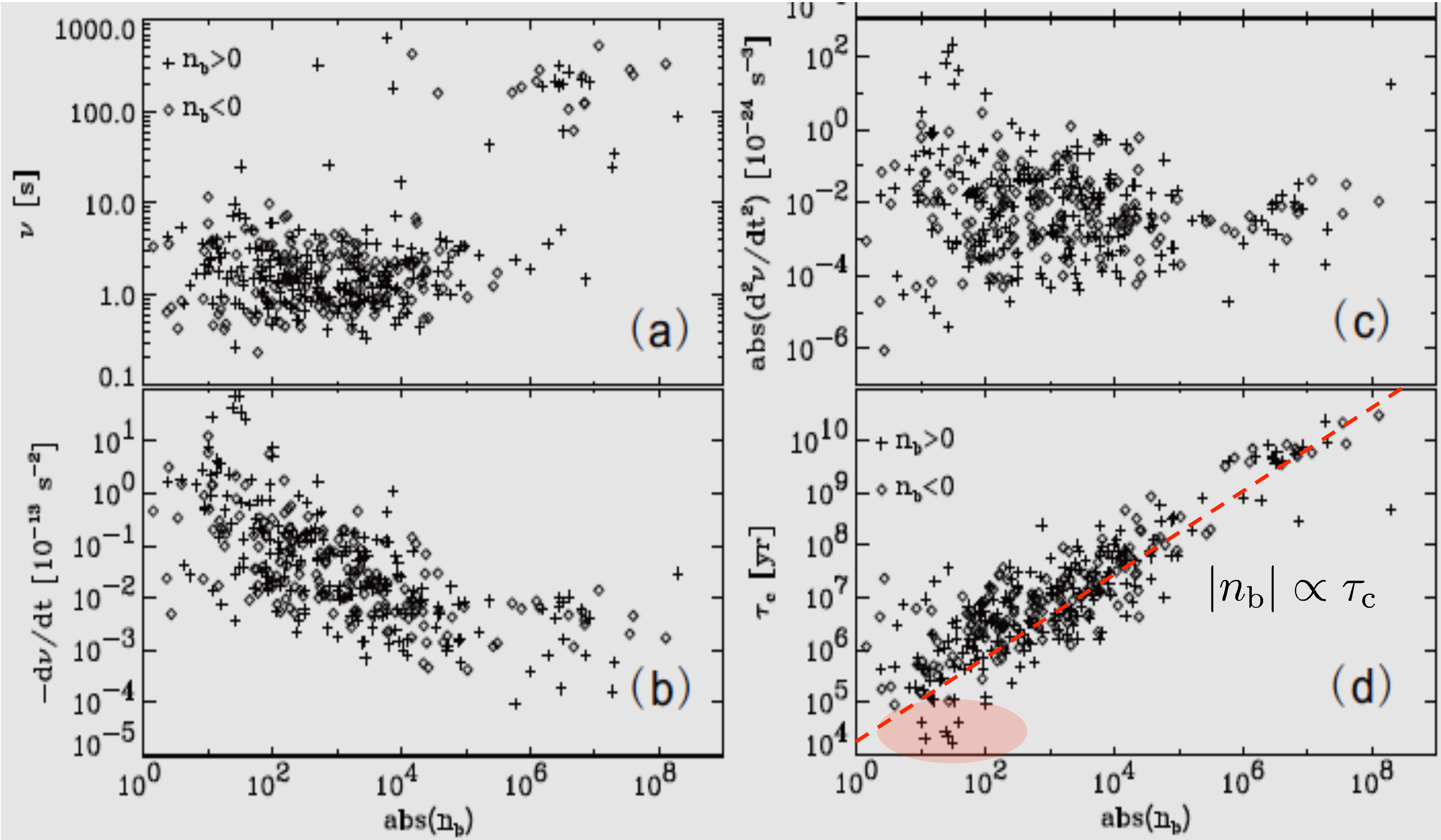


Braking law and braking index (2)

Blandford & Romani (1988) found that if assuming
$\dot{\Omega} = -K(t)\Omega^3 \Rightarrow \frac{\dot{K}}{K}\frac{\nu}{\dot{\nu}} = n_{\rm b} - 3$, $n_{\rm b} = \ddot{\nu}\nu/\dot{\nu}^2$.
We can further get: $\dot{K} = \frac{\ddot{\nu}}{\nu^3}(\frac{3}{n_{\rm b}} - 1) = \frac{\dot{\nu}^2}{\nu^4}(3 - n_{\rm b}) \Rightarrow$
$n_{\rm b} = 3 \Rightarrow \dot{K} = 0$; $n_{\rm b} > 3 \Rightarrow \dot{K} < 0$; $n_{\rm b} < 3 \Rightarrow \dot{K} > 0$

Physical meaning of the Blandford & Romani formulation:
The B-dipole radiation is responsible for the instantaneous spin-down of a pulsar with a varying $K(t) \sim$ torque.
$\Rightarrow$**This is the basis of the present work.**

Observed correlations of braking indices



$|n_\mathrm{b}| \propto \tau_\mathrm{c}$

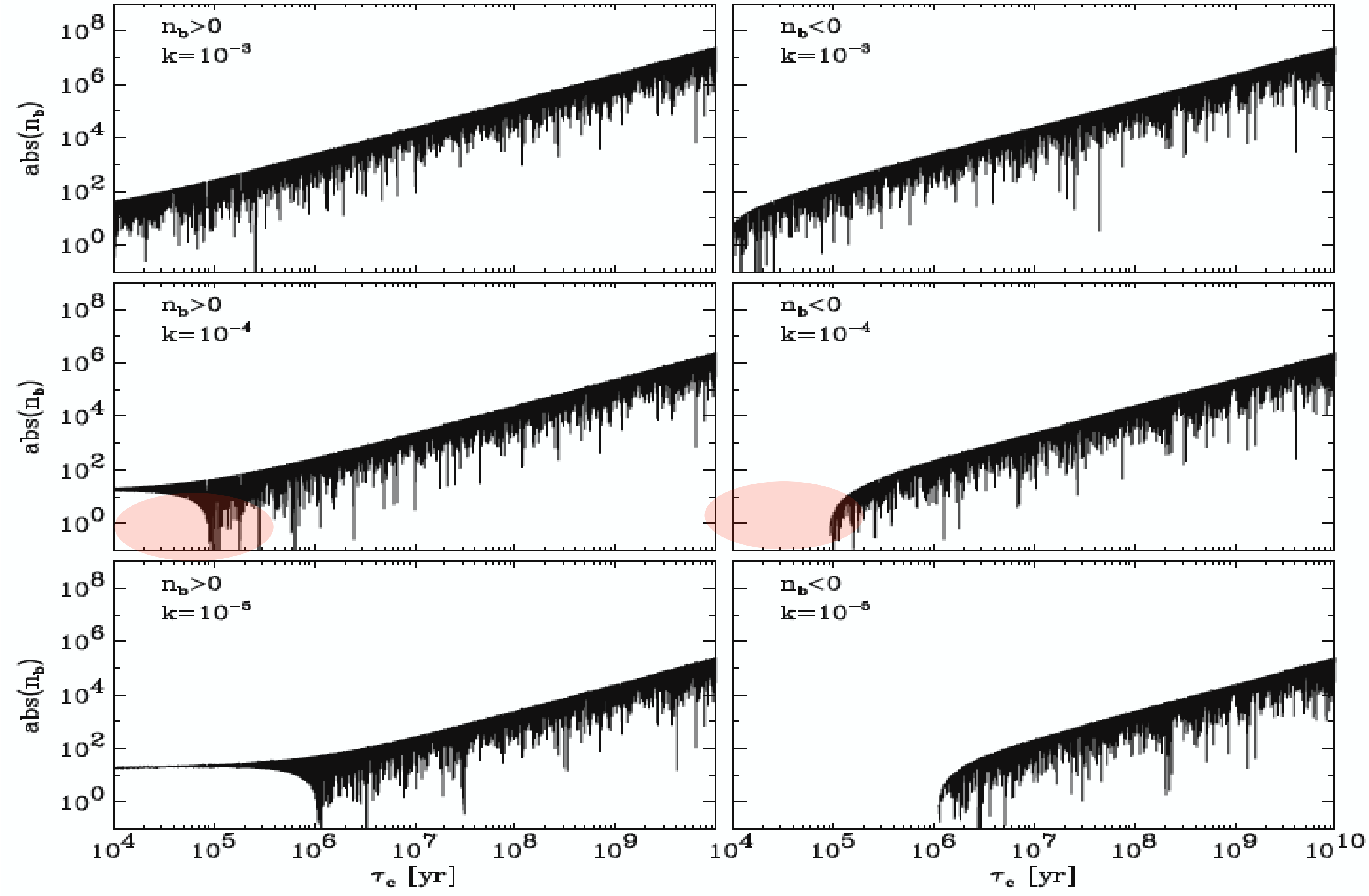
Analytical model of braking index evolution

Define $\Omega\dot{\Omega} = -AB^2\Omega^4 = -K\Omega^4 \Rightarrow \dot{\nu} = -K\nu^3 \Rightarrow$
$\nu^{-2} = \nu_0^{-2} + 2\int_0^t K dt$

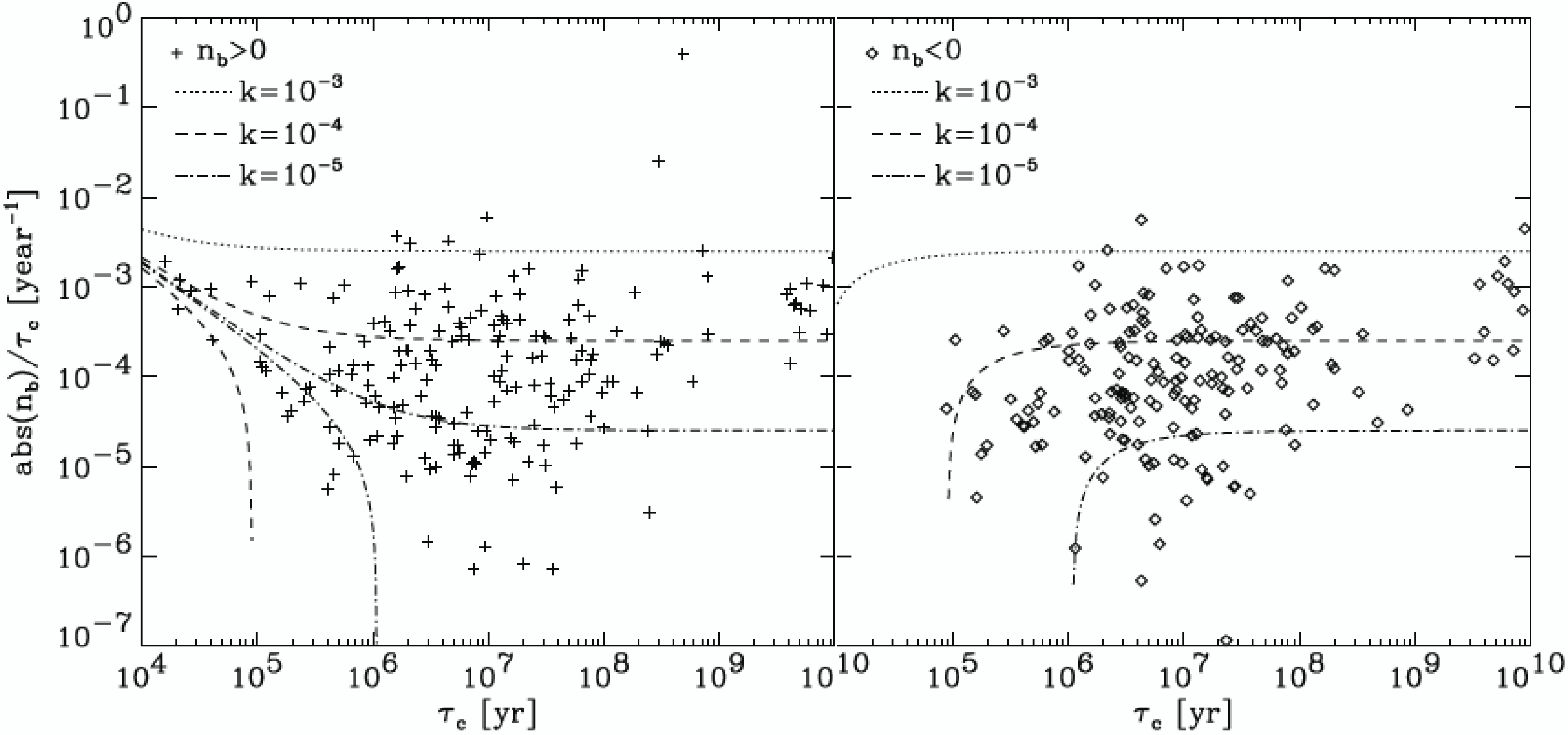
Assume the following model of B-evolution:
$B(t) = B_{\rm d}(t)(1 + k\sin(\phi + 2\pi\frac{t}{T}))$ & $B_{\rm d} = B_0(t/t_0)^{-\alpha}$
$\Rightarrow \tau_{\rm c} = -\frac{\nu}{2\dot{\nu}} = \frac{1}{2K\nu^2} \approx t\ln\frac{t}{t_0}$ (for $\alpha = \frac{1}{2}$ & $k \ll 1$)
& $n_{\rm b} = \frac{\ddot{\nu}\nu}{\dot{\nu}^2} = 3 + \ln\frac{t}{t_0}(2 - 4ftC) = 3 + \frac{\tau_{\rm c}}{t}(2 - 4ftC)$
$f = \frac{2\pi k}{T}$ & $C = \cos(\phi + 2\pi\frac{t}{T})$

Model predictions agree with data:
(1) For young pulsars: $n_{\rm b} \approx 3 + 2\ln\frac{t}{t_0} = 3 + 2\frac{\tau_{\rm c}}{t} > 0$
(2) For old pulsars: $n_{\rm b} \approx \pm 4\tau_{\rm c}f < 0$ or > 0

Analytical calculations of braking index

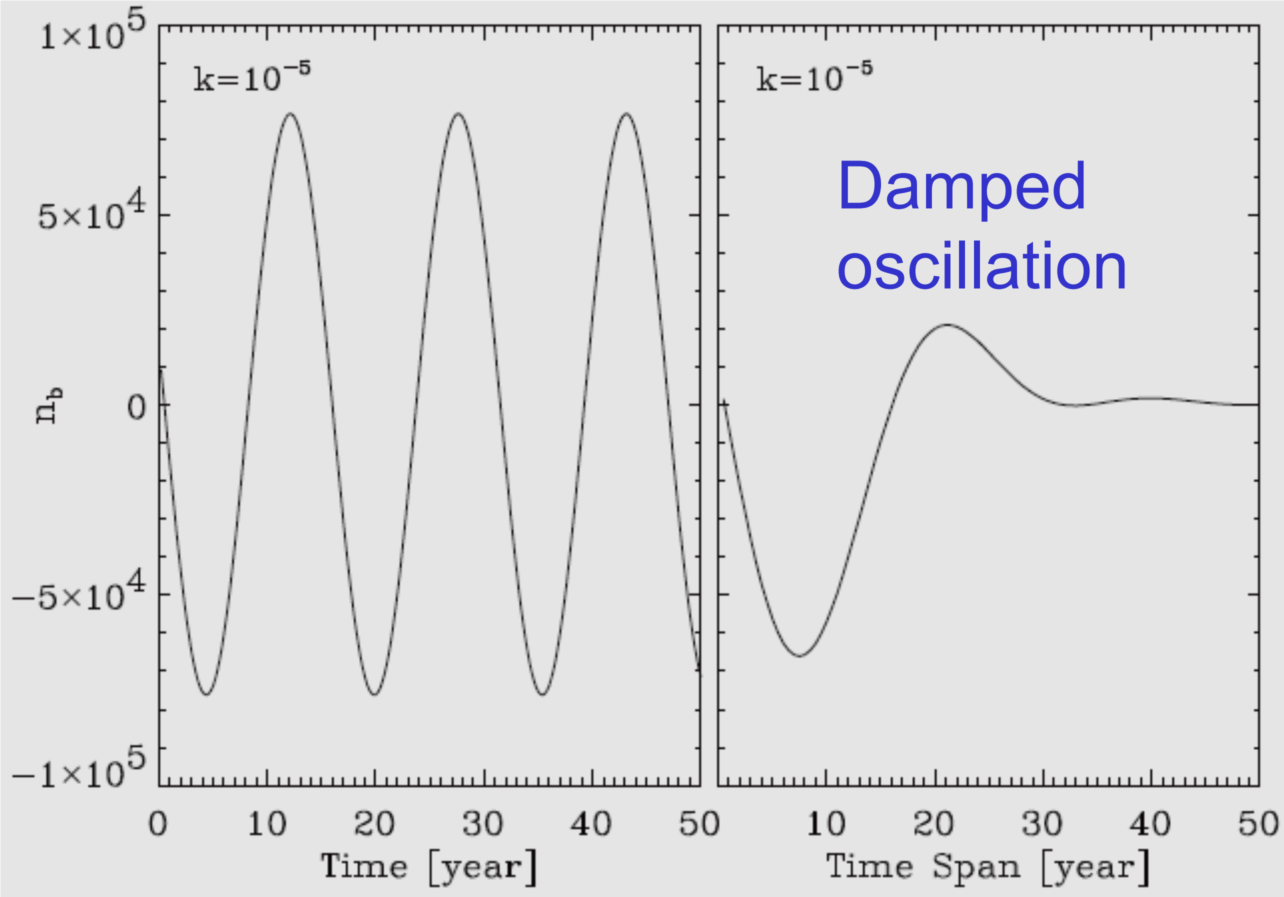


Model comparison with data



Correlation between $n_{\rm b}/\tau_{\rm c}$ and $\tau_{\rm c}$. For simplicity, we take $n_{\rm b} = 3 + \ln\frac{t}{t_0}(2 \pm 4ft) = 3 + \frac{\tau_{\rm c}}{t}(2 \pm ft), f = \frac{2\pi k}{T}, T = 10$ yr.

Our model prediction



When the time span used < the oscillation period, the observed n_b is similar to the model predicted (instantaneous) one.

It is possible to reconstruct the left panel from the right one from data.

left: Instantaneous $n_{\rm b}$ as a function of *real* time.

Right: Timing model fitted $n_{\rm b}$ vs. time *span* of the fitting:

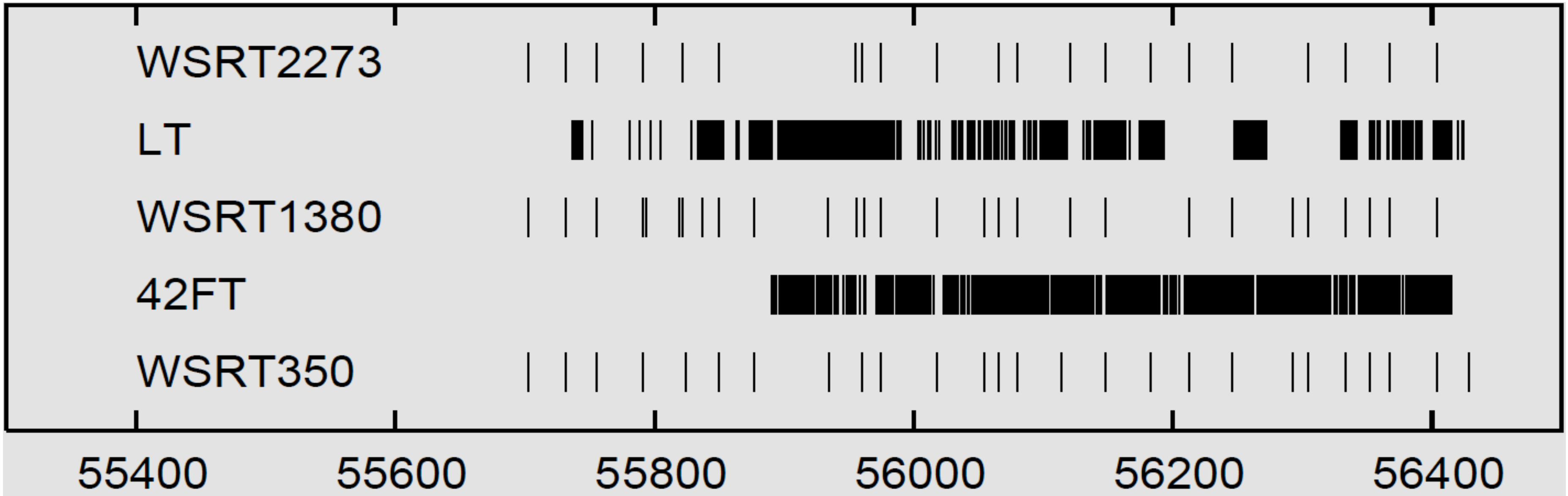
$$\Phi(t) = \Phi_0 + \nu_0(t - t_0) + \frac{1}{2}\dot{\nu}_0(t - t_0)^2 + \frac{1}{6}\ddot{\nu}_0(t - t_0)^3$$

5. Damped Oscillation of Pulsar Braking Index: the Case of PSR B1937+21 with High Cadence Observations

Shuxu Yi et al 2015, submitted.

ToA data on this pulsar from Jodrell Bank

High cadence observations of B1937+21

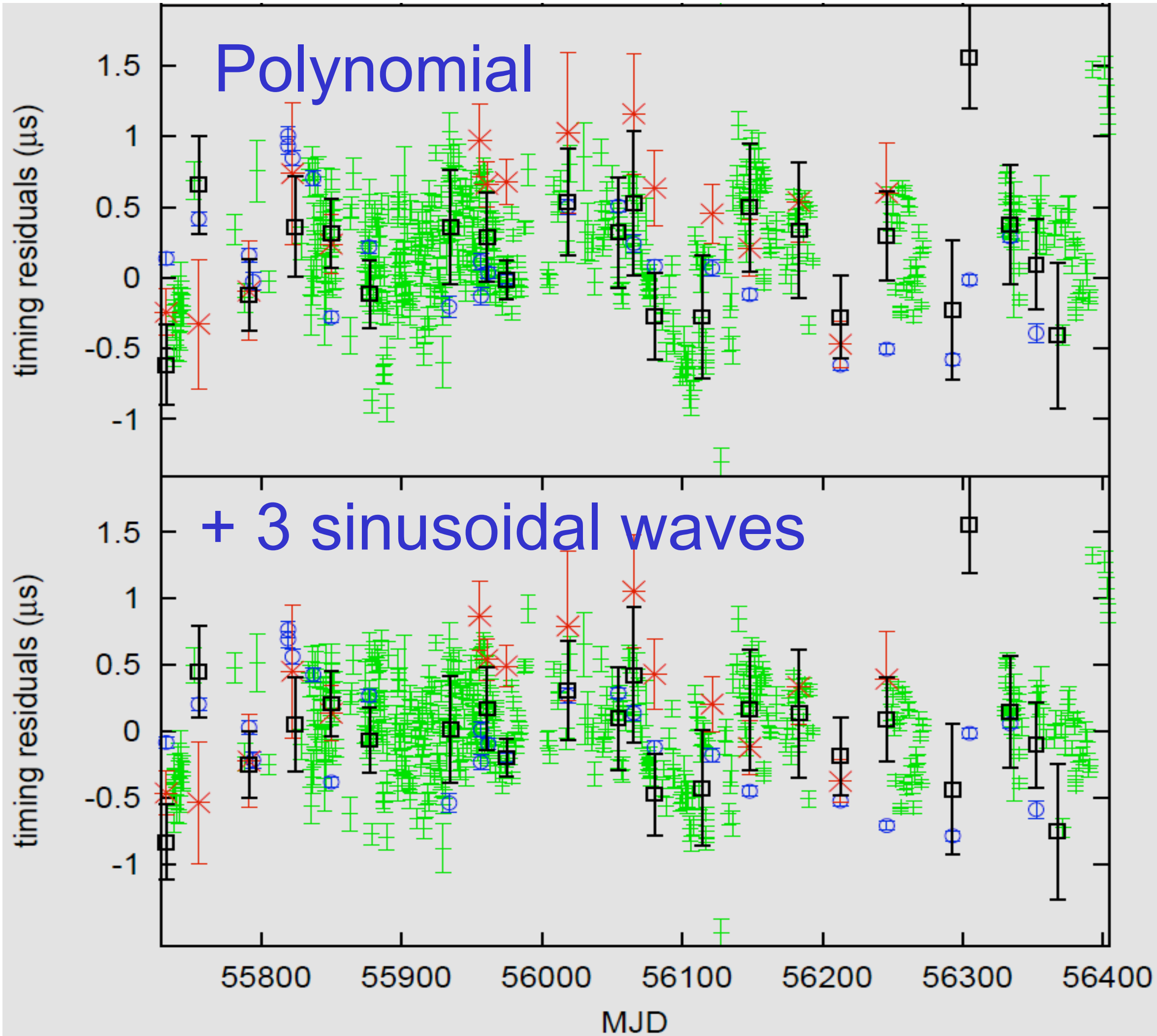


528 pulse times-of-arrival (ToAs) over the span of 650 days: typically daily or twice daily.

1. Westerbork Synthesis Radio Telescope (WSRT)
2. 42 feet telescope of Jodrell Bank (42FT)
3. Lovell telescope (LT)

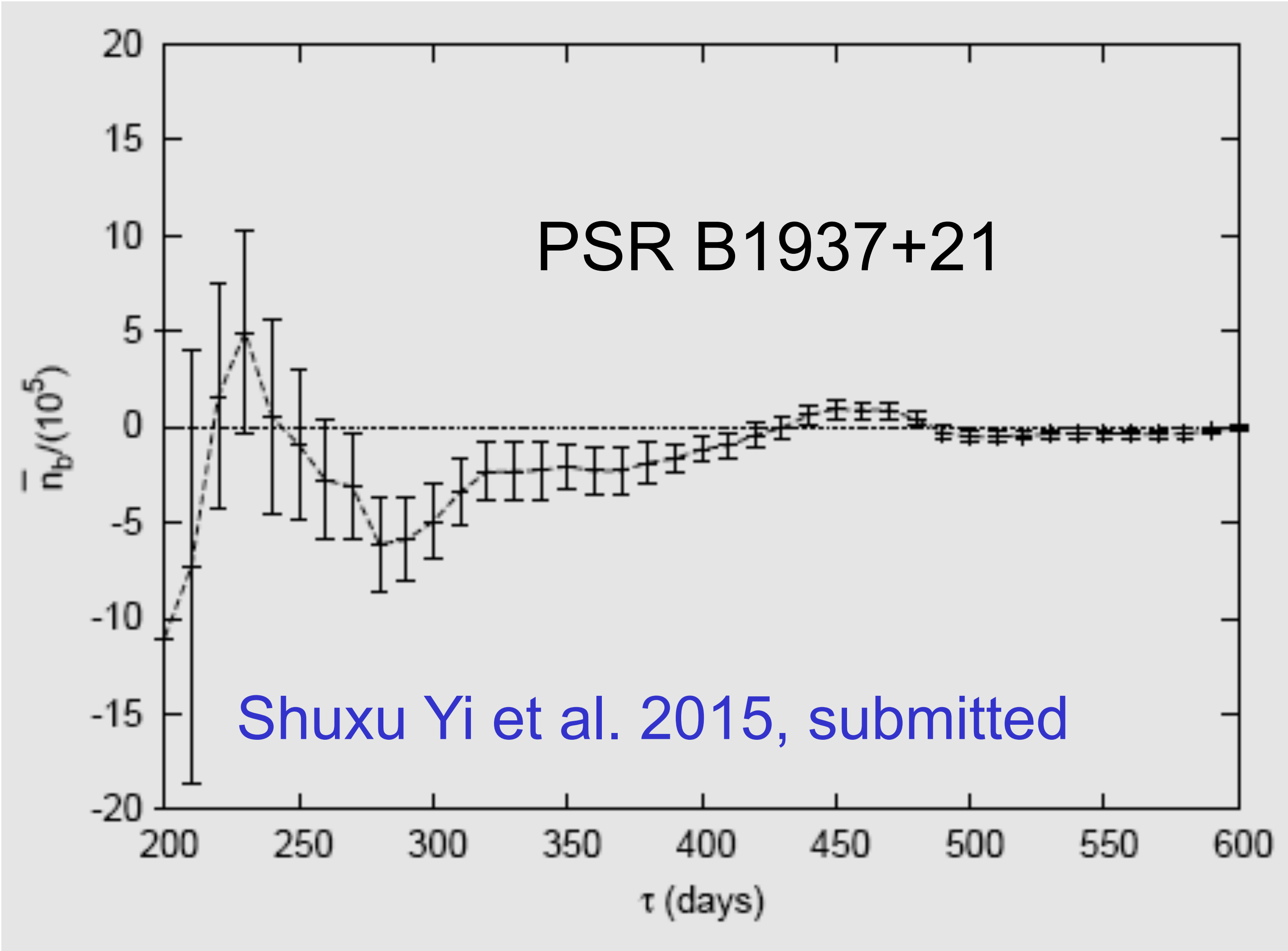
Timing residuals of B1937+21

$$\phi_{\mathrm{m}}(t) = \phi_0 + \nu_0\,(t - t_0) + \frac{1}{2}\dot{\nu}_0\,(t - t_0)^2 + \frac{1}{6}\ddot{\nu}_0\,(t - t_0)^3.$$



Model prediction confirmed!

n_b



Time span (Days)

Fit with our model of B-field evolution

$$\phi_{\rm m}(t) = \phi_0 + \nu_0 (t - t_0) + \frac{1}{2}\dot{\nu}_0 (t - t_0)^2 + \frac{1}{6}\ddot{\nu}_0 (t - t_0)^3.$$

$$\chi^2 = 15114.46 \ (dof = 478)$$

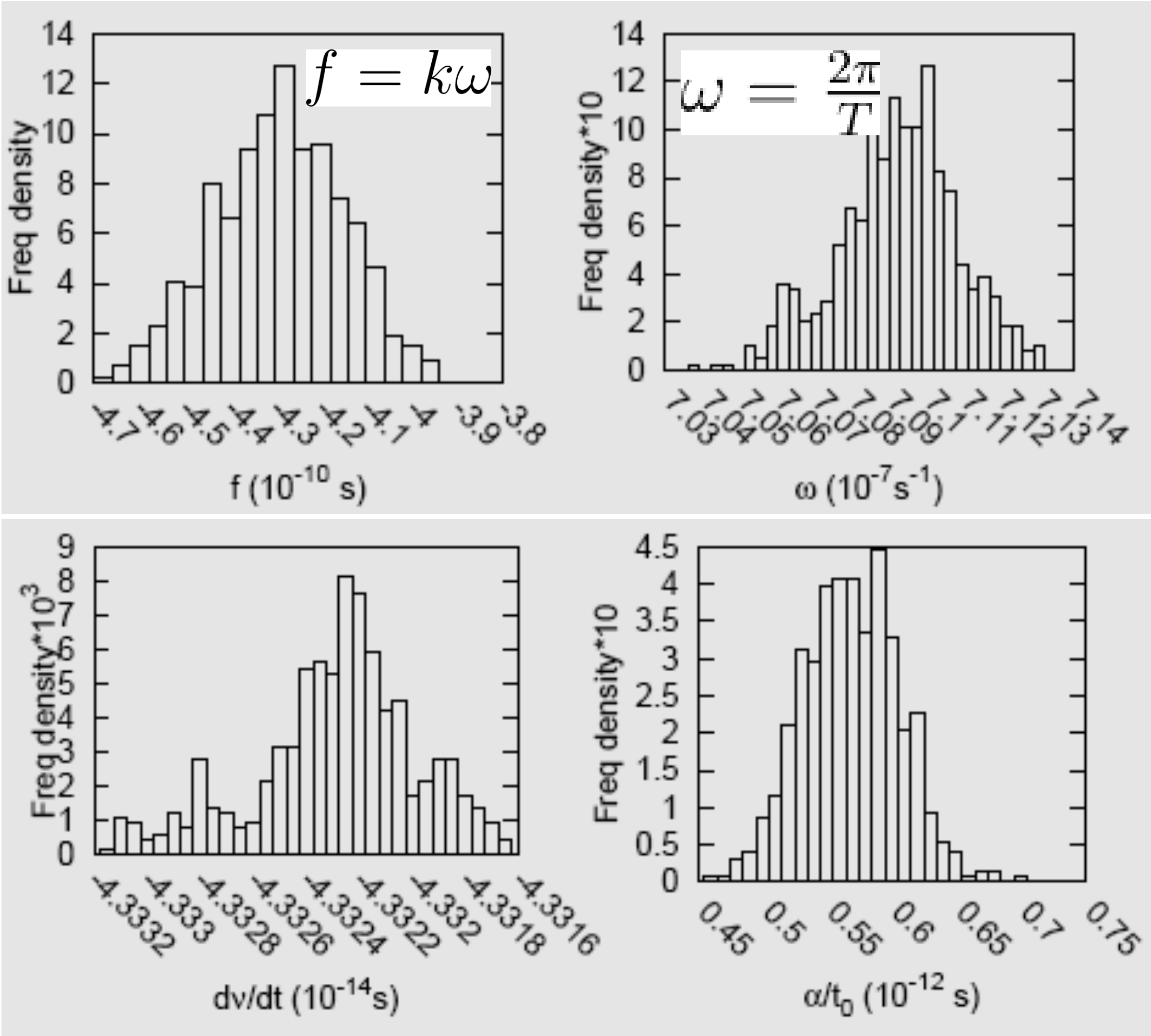
$$B(t) = B_0 \left(\frac{t}{t_0}\right)^{-\alpha} \left[1 + k \sin\left(\phi + 2\pi \frac{t}{T}\right)\right],$$

$$\ddot{\nu} = 3\dot{\nu}^2/\nu + 2\dot{\nu}\left[\frac{f\cos(\omega t + \varphi)}{1 + k\sin(\omega t + \varphi)} - \alpha/t\right]$$

$$\chi^2 = 12993.56 \ (dof = 473), \ F = 10.8, \ P = 10^{-13}$$

Our B-field evolution is preferred with high significance!

B-parameter fitting in our model



$$B(t) = B_0 \left(\frac{t}{t_0}\right)^{-\alpha} \left[1 + k \sin\left(\phi + 2\pi \frac{t}{T}\right)\right],$$

Consistent with our previous results.

2-D Correlation plot

$$B(t) = B_0 \left(\frac{t}{t_0}\right)^{-\alpha} \left[1 + k \sin\left(\phi + 2\pi \frac{t}{T}\right)\right],$$

$$f = k\omega \quad \omega = \frac{2\pi}{T}$$

ω & ϕ correlated with $\dot{\nu}$

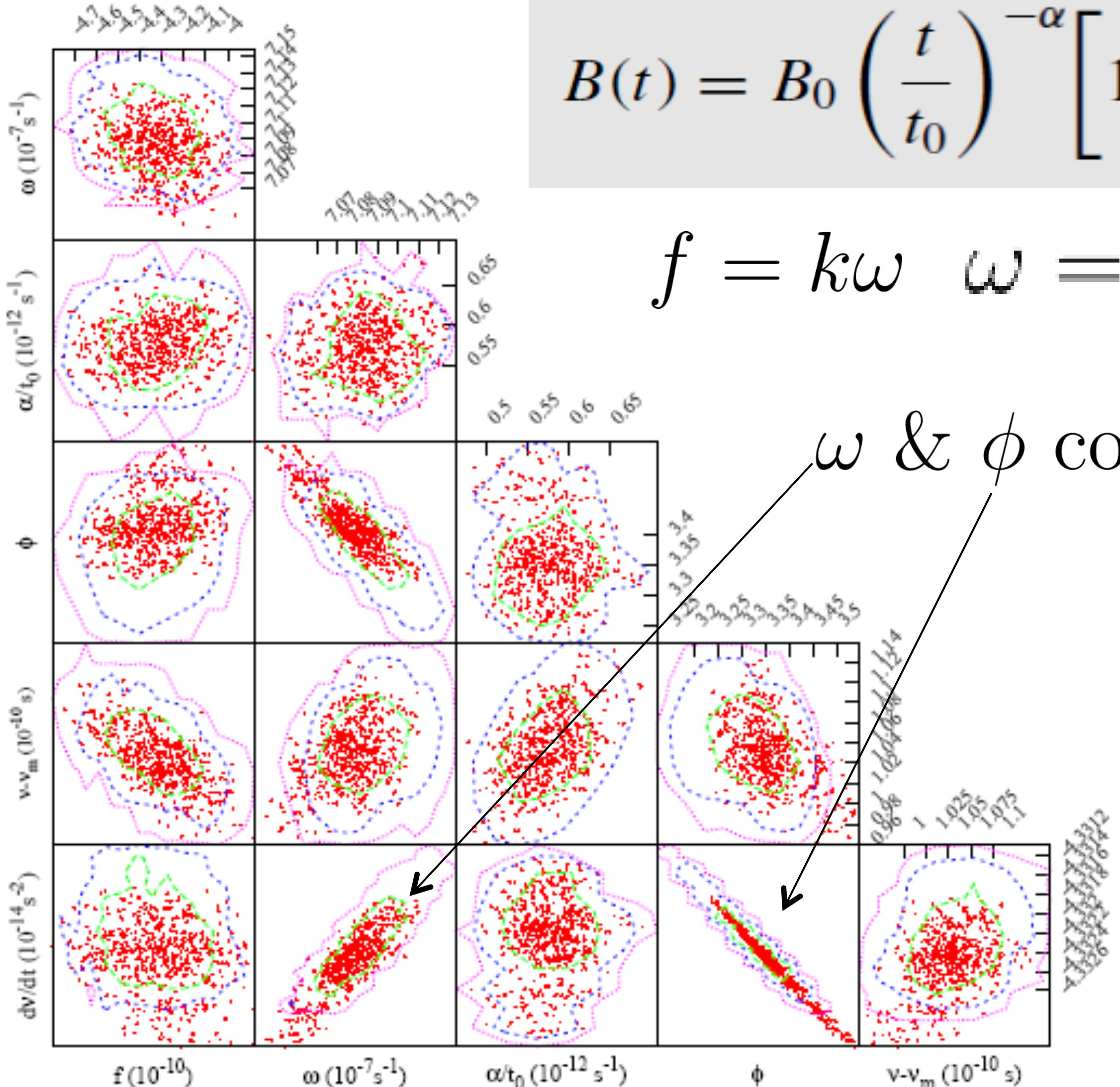
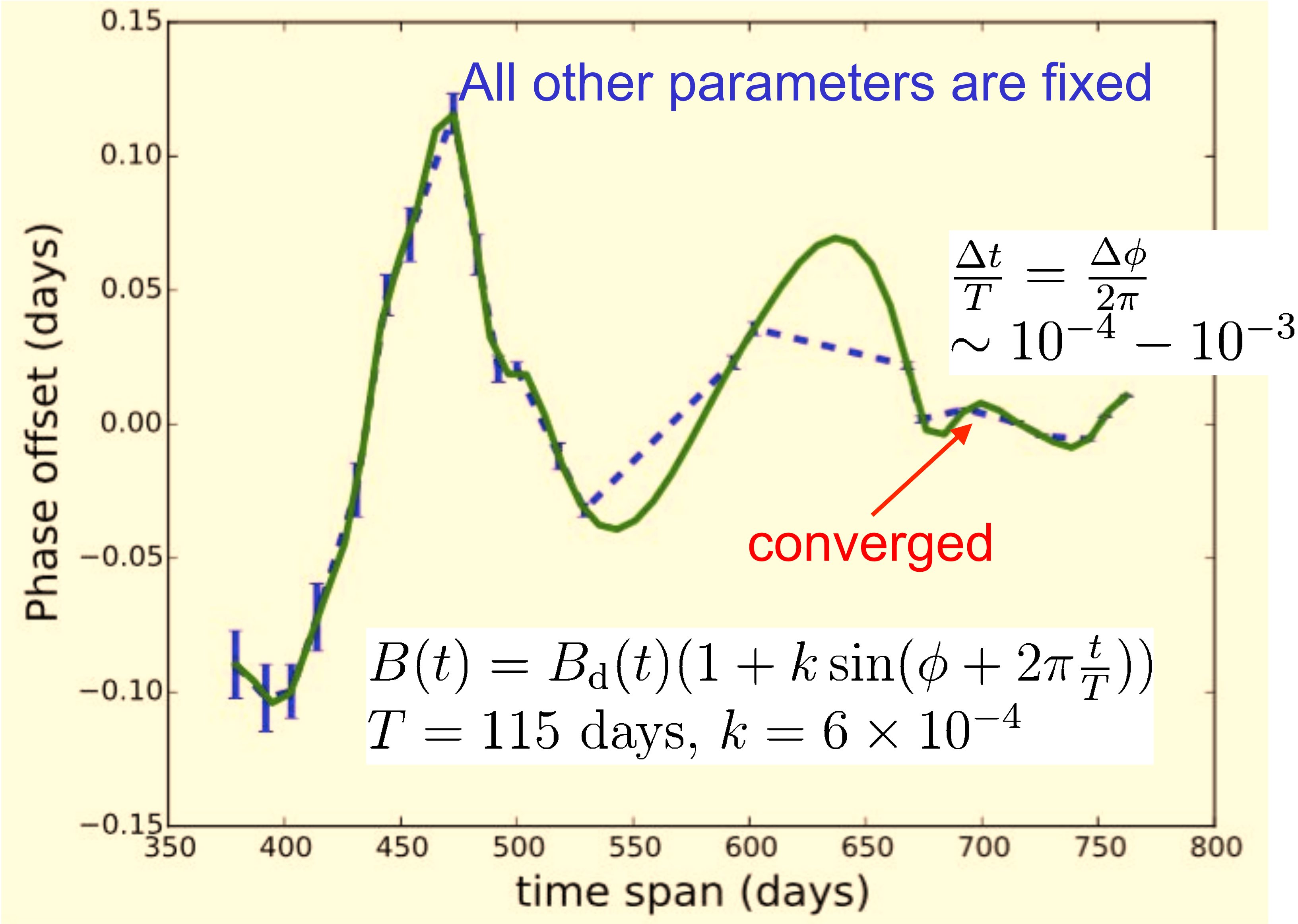
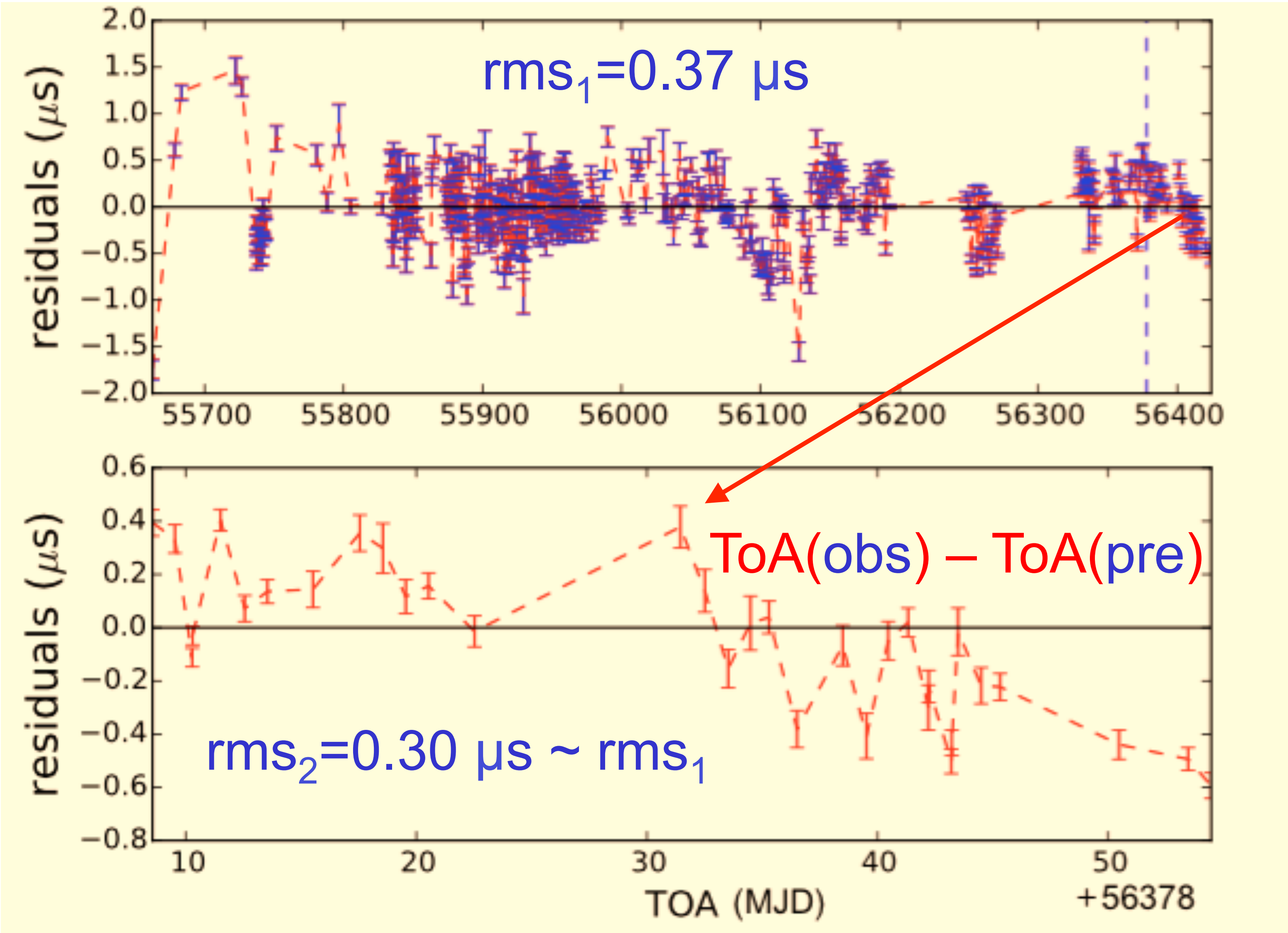


Figure 6. Two dimensional distributions of all pairs of parameters. The probability contours of one to three σ confidence are plotted on the top of each panel.

Damped-oscillation of phase of B-oscillation?



Model has predictability



We will try to do this to X-ray pulsars (need ToA data).

Answers to the questions

- How to describe the classical and slow glitches?

$$\dot{\Omega}\Omega^{-3} = -\frac{2(BR^3 \sin\chi)^2}{3c^3 I} G(t),\ \ G(t) = 1 + \kappa e^{-\Delta t/\tau},\ \ \kappa = \pm 1$$

- How to describe the timing noise of pulsars?

$$B = B_0\left(\frac{t}{t_0}\right)^{-\alpha}\left(1 + \sum K_i \sin\left(\phi_i + 2\pi\frac{t}{T_i}\right)\right),$$

- Limitations on pulsar timing for spacecraft navigation?
  - Timing noise → intrinsic accuracy on navigation
  - Proper modeling of pulsar's spin-down can reduce timing noise → better navigation accuracy

Many thanks for your attention!